Dispersion law in the periodic array of emitters

(Dated: January 15, 2024)

We are interested in finding the eigenstate ψ and the eigenfrequency ω for the infinite array of emitters:

$$\sum_{n=-\infty}^{\infty} H_{m,n}\psi_n = \omega\psi_m, \quad H_{m,n} = (\omega_0 - \mathrm{i}\gamma)\delta_{mn} - \mathrm{i}\gamma_{1\mathrm{D}}\mathrm{e}^{\mathrm{i}\omega_0 d|m-n|/c}\,. \tag{1}$$

Here, ω_0 is the resonant frequency, c is the speed of light, γ is the nonradiative decay rate, $\gamma_{1\mathrm{D}}$ is the radiative decay rate. Due to the translational symmetry of the problem, $H_{m+l,n+l} = H_{m,n}$ the solution ψ_m can be sought in the form

$$\psi_m = \psi_0\mathrm{e}^{\mathrm{i}Km}, \tag{2}$$

where K is the polariton wave vector depeding on ω.

Goal: substitute Eq. (2) into Eq. (1) and find the equation for $K(\omega)$ describing the law $\omega(K)$. Plot $K(\omega)$ numerically in the range $\omega_0 - 10\gamma_{1\mathrm{D}} < \omega < \omega_0 + 10\gamma_{1\mathrm{D}}$ for the following set of parameters: $\gamma = 0, \omega_0 d/c = 0.5$.

Answer:

$$\cos K = \cos\tfrac{\omega_0 d}{c} - \frac{\gamma_{1\mathrm{D}}\sin\tfrac{\omega_0 d}{c}}{\omega_0 - \omega - \mathrm{i}\gamma}\,. \tag{3}$$

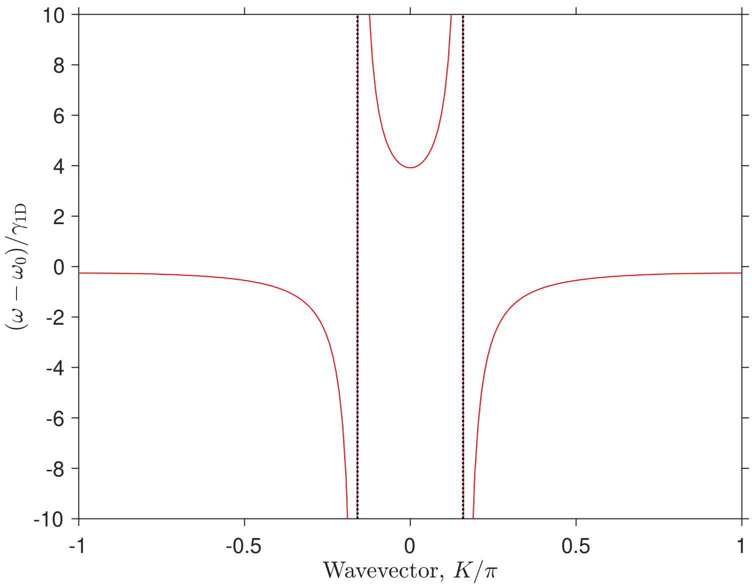

FIG. 1 Dispersion law in the array of emitters.