Dispersion law in the periodic array of chiral emitters

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We are interested in finding the eigenstate ψ and the eigenfrequency ω for the infinite array of emitters, chirally coupled to the waveguide:

$$\sum_{n=-\infty}^{\infty} H_{m,n}\psi_n = \omega\psi_m, \quad H_{m,n} = (\omega_0 - \mathrm{i}\gamma)\delta_{mn} - \mathrm{i}\mathrm{e}^{\mathrm{i}\omega_0 d|m-n|/c} \times \begin{cases} \gamma_{\rightarrow}, & m > n\,, \\ ,\gamma_{1\mathrm{D}}, & m = n\,, \\ \gamma_{\leftarrow}, & m < n\,. \end{cases} \tag{1}$$

Here, ω_0 is the resonant frequency, c is the speed of light, γ is the nonradiative decay rate, $\gamma_{1\mathrm{D}} = (\gamma_{\leftarrow} + \gamma_{\rightarrow})/2$ is the radiative decay rate of the atom into the waveguide,

$$\gamma_{\rightarrow} = \frac{2}{1+\xi}\gamma_{1\mathrm{D}}, \quad \gamma_{\leftarrow} = \frac{2\xi}{1+\xi}\gamma_{1\mathrm{D}} \tag{2}$$

are the forward and backward emission rates ($0 \le \xi \le 1$). Due to the translational symmetry of the problem, $H_{m+l,n+l} = H_{m,n}$ the solution ψ_m can be sought in the form

$$\psi_m = \psi_0 \mathrm{e}^{\mathrm{i}Km}, \tag{3}$$

where K is the polariton wave vector depending on ω.

Goal: substitute Eq. (3) into Eq. (1) and find the equation for $K(\omega)$ describing the law $\omega(K)$. Plot on the same graph the curves $K(\omega)$ in the range $\omega_0 - 10\gamma_{1\mathrm{D}} < \omega < \omega_0 + 10\gamma_{1\mathrm{D}}$ for the following set of parameters: $\gamma = 0, \omega_0 d/c = 0.5$ for $\xi = 1, 0.5, 0.1$.

Answer:

$$\omega(K) = \omega_0 - \mathrm{i}\gamma + \gamma_{1\mathrm{D}}\frac{\sin\varphi + \chi\sin K}{\cos K - \cos\varphi}, \quad \chi = \frac{1-\xi}{1+\xi}, \quad \varphi = \frac{\omega_0 d}{c}\,. \tag{4}$$

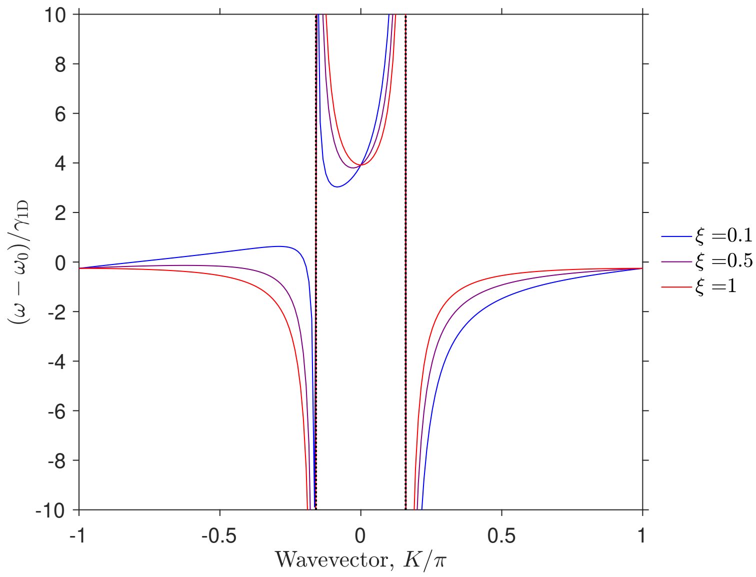

FIG. 1 Dispersion law in the chiral array of emitters for 3 chirality degrees ξ.