Nonconjugated orthogonality for non-Hermitian Hamiltonian

(Dated: August 16, 2023)

Consider an eigenvalue problem

$$\sum_{n=1}^{N} H_{m,n}^{(\nu)} \psi_n^{(\nu)} = \omega^{(\nu)} \psi_m^{(\nu)}, \tag{1}$$

with symmetric complex non-Hermitian $N \times N$ Hamiltonian matrix H, such that $H_{m,n} = H_{n,m} \neq H_{m,n}^*$. Such non-Hermitian but symmetric Hamiltonians are ubiquitos for the problems where emitters are coupled to propagating photons.

Goal. Prove that the eigenvectors $\psi_m^{(\nu)}$ satisfy the non-conjugated orthogonality relationship

$$\sum_{m=1}^{N} \psi_m^{(\nu)} \psi_m^{(\mu)} = 0 \text{ if } \omega_\mu \neq \omega_\nu \,. \tag{2}$$