Light reflection and transmission from a resonant emitter

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We study scattering of an electromagnetic wave, propagating in a one-dimensional waveguide, on a resonant point light emitter, see Fig. 1. We describe light-emitter interaction by a wave equation

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}E(z) + q^2 E(z) = -4\pi q^2 p\delta(z), \tag{1}$$

where

$$p = \frac{a}{\omega_0 - \omega - \mathrm{i}\gamma}E(0)\,, \tag{2}$$

characterizes the resonant polarization of the emitter. Here, $E(z)$ is the amplitude of the electric field at the frequency ω, $q = \omega/c$ is the light wave vector, ω_0 is the resonant frequency of the emitter, γ is the phenomenological decay rate, characterizing the nonradiative processes withing the emitter. An electromagnetic wave with the amplitude E_0 is incident from the left and is scattered on the emitter, which means that

$$E(z) = \begin{cases} E_0(\mathrm{e}^{\mathrm{i}qz} + r\mathrm{e}^{-\mathrm{i}qz}), & z < 0 \\ E_0 t\mathrm{e}^{\mathrm{i}qz}, & z > 0\,. \end{cases} \tag{3}$$

Here, r and t are the light reflection and transmission coefficients.

Goal 1: Prove, that reflection and transmission coefficients can be presented in the form

$$r(\omega) = \frac{\mathrm{i}\gamma_{1\mathrm{D}}}{\omega_0 - \omega - \mathrm{i}(\gamma_{1\mathrm{D}} + \gamma)}\,, t(\omega) = \frac{\omega_0 - \omega - \mathrm{i}\gamma}{\omega_0 - \omega - \mathrm{i}(\gamma_{1\mathrm{D}} + \gamma)}\,. \tag{4}$$

Find the expression for the parameter $\gamma_{1\mathrm{D}}$, that characterizes spontaneous decay rate of the emitter in the waveguide.

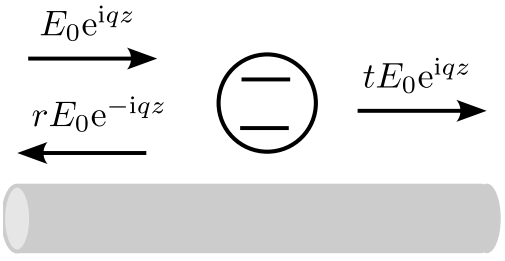

FIG. 1 Schematics of light reflection and transmission from a scatterer.

Goal 2: Calculate the quantity $1 - |r^2(\omega)| - |t(\omega)^2|$ for $\gamma = 0$ and for arbitrary real γ. Explain, what is the physical meaning of this quantity.