Dispersion equation in a periodic array of resonant scatterers.

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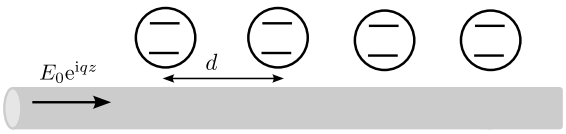

FIG. 1 Periodic array of resonant scatterers coupled to the waveguide.

We consider wave propagation in a one-dimensional periodic array of scatterers, shown in Fig. 1. Wave scattering on each period is characterized by the 2×2 transfer matrix

$$T = T_{\text{free}} T_{\text{res}} \tag{1}$$

where

$$T_{\text{free}} = \begin{pmatrix} e^{iqd} & 0 \\ 0 & e^{-iqd} \end{pmatrix} \tag{2}$$

is the transfer matrix for the free waveguide propagation in the basis of propagating waves, $q = \omega/c$, and

$$T_{\text{res}} = \frac{1}{t} \begin{pmatrix} t^2 - r^2 & r \\ -r & 1 \end{pmatrix}, \quad r = \frac{i\gamma_{1D}}{\omega_0 - \omega - i(\gamma_{1D} + \gamma)}, \quad t = 1 + r. \tag{3}$$

is the resonant matrix of a scatterer.

We are interested in the propagating Floquet-Bloch solutions, that satisfy $T\psi = e^{iK}\psi$.

Goal: Find the dispersion equation, that is express $\cos K$ via the transfer matrix elements.

Hint: (Ivchenko, 1991)

References

Ivchenko, E. L., 1991, Sov. Phys. Sol. State **33**(8), 1344.