Bragg reflection of light from quantum-well structures

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Resonant light reflection and transmission spectra for are calculated for heterostructures with a finite system of equidistant quantum wells. Recurrence relations connecting amplitude reflection coefficients for systems of N and $N-2$ quantum wells are derived. The analytical properties of the reflection coefficient as a function of the complex frequency ω are analyzed. A method is proposed for taking into account the mirror symmetry of a system of N quantum wells and permitting one to find the complex frequencies of mixed exciton-photon unstationary excitations, or exciton polaritons. The resonant Bragg structures are shown to be a special case, where only one of the N eigenmodes is radiative.

INTRODUCTION

Light wave propagation in a layered medium is a classical problem in solid state optics.[1–3] Comprehensive research performed for many years in this area has culminated not only in developing complex multilayered structures with remarkable reflection and antireflection properties having an application potential, but in observing some specific optical phenomena in such structures as well. This work develops a theory of light reflection and transmission near the exciton resonance frequency in a structure with a system of equidistant quantum wells, where the optical thickness D between the neighboring wells is comparable to the wavelength λ in vacuum. Particular attention is focused on the Bragg reflection, where the condition $D=\lambda/2$ is met. Earlier studies dealt with resonant reflection from single quantum wells[4–6] or short-period heterostructures with $D/\lambda \ll 1$.[7–10]

1. GENERAL EXPRESSIONS FOR THE REFLECTION AND TRANSMISSION COEFFICIENTS

Let us consider a heterostructure containing N quantum wells separated by a distance d (Fig. 1). The reflection coefficient for this structure can be appropriately written

$$R=\left|\frac{r_{01}+\tilde{r}_N e^{2i\phi'}}{1+r_{01}\tilde{r}_N e^{2i\phi'}}\right|^2, \tag{1}$$

where r_{01} is the amplitude reflection coefficient at the interface between vacuum and the outermost barrier layer, $\tilde{r}_N$ is the coefficient of reflection inside the structure from the system of N quantum wells, i.e. the amplitude ratio of the reflected and transmitted waves at the plane shifted by $d/2$ from the center of the left-hand well, and ϕ' is the phase change a wave undergoes in travelling a distance l in the barrier (see Fig. 1).

The quantity $\tilde{r}_N$ can be conveniently calculated by means of transfer matrices relating the forward and backward waves at the left- and right-hand boundaries of a layer.[9] Selecting for the right-hand boundary of the structure a plane set at $d/2$ to the right of the outermost well, we find that the transfer matrix $\hat{T}^{(N)}$ involving N wells is equal to that for one well,

$$\hat{T}_1=\frac{1}{\tilde{t}_1}\begin{bmatrix}\tilde{t}_1^2-\tilde{r}_1^2 & \tilde{r}_1\\ -\tilde{r}_1 & 1\end{bmatrix} \tag{2}$$

raised to Nth power, $\hat{T}_1^N$. At normal incidence the reflection and transmission coefficients for a single quantum well in the vicinity of the exciton resonance can be written[6,8]

$$\tilde{t}_1=e^{ikd}t_1, \quad \tilde{r}_1=e^{ikd}r_1,$$

$$t_1=1+r_1, \quad r_1=\frac{i\Gamma_0}{\omega_0-\omega-i(\Gamma+\Gamma_0)}, \tag{3}$$

where $k=(\omega/c)\sqrt{\varepsilon_b}$, ε_b is the barrier permittivity, ω_0 is the resonant frequency renormalized with inclusion of the exciton-photon coupling, and Γ_0 and Γ are, respectively, the radiative and nonradiative exciton damping constants in a single quantum well. Equation (3) neglects the difference between ε_b and the background permittivity in the quantum well layer. Within a resonance spectral interval narrow compared to ω_0 we may set in Eq. (3) $k=(\omega_0/c)\sqrt{\varepsilon_b}$ and consider it a constant.

The homogeneous nonradiative broadening of the reflectance spectra is due to exciton scattering by heterostructure defects and phonons. However, when describing optical spectra, the quantity Γ in Eq. (3) is frequently understood to be the sum of two contributions, $\Gamma_h+\Gamma_{inh}$, the first of them being related to the intrinsic homogeneous broadening, and the second taking effectively into account the inhomogeneous broadening of the exciton resonant frequency. Under certain conditions Γ_h and Γ_{inh} can act as independent parameters. As an illustration, let us consider the amount of energy absorbed in a single quantum well. If there is no inhomogeneous broadening, it can be written

$$w(\omega)=1-|r_1|^2-|t_1|^2=\frac{2\Gamma_h\Gamma_0}{(\omega_0-\omega)^2+(\Gamma_h+\Gamma_0)^2}.$$

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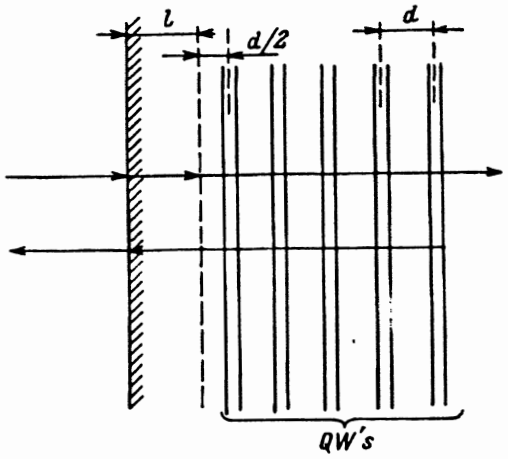

FIG. 1. Geometry of light reflection from a structure with five quantum wells (QW's).

We introduce a distribution function $P(\omega_0)$ describing the large-scale inhomogeneous broadening of the exciton resonant frequency in the well plane. Then for the fraction of absorbed energy averaged over the spot width we can write

$$\bar{w}(\omega)=\int d\omega_0 P(\omega_0)w(\omega).$$

In the very simple case of the Lorentzian distribution when

$$P(\omega_0)=\frac{1}{\pi}\frac{\Gamma_{\text{inh}}}{(\omega_0-\tilde{\omega}_0)^2+\Gamma_{\text{inh}}^2},$$

we obtain

$$\bar{w}(\omega)=\frac{\Gamma_h}{\Gamma_h+\Gamma_0}\frac{2(\Gamma+\Gamma_0)\Gamma_0}{(\tilde{\omega}_0-\omega)^2+(\Gamma+\Gamma_0)^2},$$

where $\Gamma=\Gamma_h+\Gamma_{\text{inh}}$.

The calculation of $\tilde{r}_N$ and $\tilde{t}_N$ in Ref. 9 involved determination of the eigenvalues and eigenvectors of the matrix $\hat{T}_1$. We shall use here a different, though equivalent, method based on the fact that all powers $\hat{T}_1^N$, irrespective of the power, have the same matrix structure

$$\hat{T}_1^N=S_n\hat{I}+Q_N\left[\left(F-\frac{r_1}{t_1}\right)\hat{\sigma}_z+i\frac{r_1}{t_1}\hat{\sigma}_y\right], \tag{4}$$

where

$$F=(1+\cos kd)\frac{r_1}{t_1}+i\sin kd, \tag{5}$$

$\hat{I}$ is the 2×2 identity matrix, and $\hat{\sigma}_y$ and $\hat{\sigma}_z$ are standard Pauli matrices introduced here for conciseness. The coefficients S and Q with subscripts N and $N-1$ are connected through a matrix recursion relation

$$\begin{pmatrix}S_N\\Q_N\end{pmatrix}=\hat{L}\begin{pmatrix}S_{N-1}\\Q_{N-1}\end{pmatrix}=\hat{L}^{N-1}\begin{pmatrix}S_1\\Q_1\end{pmatrix}, \tag{6}$$

where

$$\hat{L}=\begin{pmatrix}G & G^2-1\\1 & G\end{pmatrix}$$

$$G=G(\omega)=\cos kd-\frac{\Gamma}{\omega-\omega-i\Gamma}\sin kd.$$

Note that the wave vector q of a light wave propagating in an infinite periodic quantum well structure satisfies the dispersion relation[8]

$$\cos qd=G(\omega). \tag{8}$$

A comparison of matrix (2) with matrix (4) for $N=1$ yields

$$S_1=G,\quad Q_1=1. \tag{9}$$

One can readily verify that

$$\hat{L}\begin{bmatrix}\pm\sqrt{G^2-1}\\1\end{bmatrix}=\varkappa_{\pm}\begin{bmatrix}\pm\sqrt{G^2-1}\\1\end{bmatrix}, \tag{10}$$

with the eigenvalues

$$\varkappa_{\pm}=G\pm\sqrt{G^2-1}. \tag{11}$$

We now decompose the column with components (9) into eigenvectors (10) and act on the linear combination thus obtained with matrix L^{N-1} to derive finally expressions for S_N and Q_N:

$$S_N=\frac{1}{2}(\varkappa_+^N+\varkappa_-^N),\quad Q_N=\frac{\varkappa_+^N-\varkappa_-^N}{\varkappa_+-\varkappa_-}. \tag{12}$$

The coefficients $\tilde{r}_N$ and $\tilde{t}_N$ are related to S_N and Q_N through

$$\tilde{r}_N=\frac{Q_N\frac{r_1}{t_1}}{S_N+Q_N\left(\frac{r_1}{t_1}-F\right)},\quad \tilde{t}_N=\frac{S_N^2-(G^2-1)Q_N^2}{S_N+Q_N\left(\frac{r_1}{t_1}-F\right)}. \tag{13}$$

Equations (13) can be used to calculate $\tilde{r}_N$ and $\tilde{t}_N$.

2. BRAGG REFLECTION

For a structure with $kd=\pi$ (or $D=\lambda/2$, where $D=\sqrt{\varepsilon_b}d$) we have $F=0$, $G=-1$, $\varkappa_+=\varkappa_-=-1$, $S_N=(-1)^N$, and $Q_N=N(-1)^{N-1}$, so that $Q_N/S_N=-N$. Substituting into Eq. (13) the ratio [see Eq. (3)]

$$\frac{r_1}{t_1}=\frac{i\Gamma_0}{\omega_0-\omega-i\Gamma},$$

we obtain

$$\tilde{r}_N(kd=\pi)=\frac{-iN\Gamma_0}{\omega_0-\omega-i(\Gamma+N\Gamma_0)}, \tag{14a}$$

$$\tilde{t}_N(kd=\pi)=(-1)^N\frac{\omega_0-\omega-i\Gamma}{\omega_0-\omega-i(\Gamma+N\Gamma_0)}. \tag{14b}$$

Thus for $kd=\pi$ the expression for $\tilde{r}_N$ differs from $\tilde{r}_1$ by Γ_0 being replaced by $N\Gamma_0$. The physical significance of this result will be discussed after a consideration of the analytic properties of $\tilde{r}_N$ as a function of the complex variable ω. We may point out, however, that the quality of quantum wells has been improving with continuing progress in growth technology. While initially the nonradiative damping Γ was con-

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3. TRANSITION TO THE LIMITING CASE OF THE APPROXIMATION OF A HOMOGENEOUS MEDIUM

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[illegible] $=2\Gamma_0/(kd)$.

[illegible] one may assume that a plane light wave[illegible] vector $q = (\omega/c)\sqrt{\varepsilon(\omega)}$ propagates in a layer [illegible] thickness Nd containing quantum wells, the amplitude coefficient of reflection from this layer being

$$\tilde{r}_N = \frac{\tilde{r}(1-e^{2i\Phi_N})}{1-\tilde{r}^2 e^{2i\Phi_N}},$$

where $\tilde{r}=(k-q)/(k+q)$, $\Phi_N=qNd$. The same result is obtained by starting from the exact expression (13) for $\tilde{r}_N$, when one takes into account that if kd, $|q|d \ll 1$ the following approximate relations are valid:

$$\sqrt{G^2-1} \approx iqd, \quad \varkappa_\pm 1 \pm iqd,$$

$$F \approx 2\frac{r_1}{t_1} + ikd, \quad \frac{r_1}{t_1} \approx \frac{id}{2k}(q^2-k^2),$$

$$\left(\frac{\varkappa_+}{\varkappa_-}\right) = \left(1+\frac{2\sqrt{G^2-1}}{G-\sqrt{G^2-1}}\right) \approx (1+2iqd)^N \approx e^{2iqNd},$$

$$\frac{Q_N}{S_N} \approx \frac{i}{qd}\frac{1-e^{2i\Phi_N}}{1+e^{2i\Phi_N}}.$$

4. INCLUSION OF MIRROR SYMMETRY OF THE HETEROSTRUCTURE

The mirror symmetry of a system of N identical quantum wells suggests a representation for $\tilde{r}_N$ and $\tilde{t}_N$ which is more appropriate for studying the analytical properties of these coefficients:

$$\tilde{r}_N = \frac{1}{2}(r_N^s + r_N^a), \quad \tilde{t}_N = \frac{1}{2}(r_N^s - r_N^a). \tag{15}$$

Here r_N^s, r_N^a are the reflection coefficients for even and odd boundary conditions for the case where plane waves with amplitudes, respectively, of the same or opposite signs fall simultaneously from the left and right on the structure. This approach was used earlier in an analysis of spectra of electron transmission through a heterostructure with mirror symmetry.[12]

For brevity, we denote in Eq. (3) the phase factor $\exp(ikd) \equiv \eta$, and introduce a dimensionless frequency $y=(\omega-\omega_0-i\Gamma)/\Gamma_0$. Since N quantum wells may be considered as $N-2$ wells surrounded by one well each on the left and right, we may write the following recursion relations

$$r_N^\gamma = \frac{T_{11}r_{N-2}^\gamma - T_{21}}{T_{22}-T_{12}r_{N-2}^\gamma}, \tag{16}$$

where $\gamma=s$ or a, and T_{ij} are the components of the matrix $\hat{t}_1$. As follows from Eq. (3)

$$r_1^s = \eta(1+2r_1), \quad r_1^a = -\eta. \tag{17}$$

Straightforward calculation shows Eq. (16) to be applicable to the case of two quantum wells as well, if one sets $r_0^s=1$, $r_0^a=-1$, i.e. $\tilde{r}_0=0$, $\tilde{t}_0=1$, as in the absence of quantum wells.

5. ANALYTICAL PROPERTIES OF THE REFLECTION COEFFICIENT. TRANSITION TO THE LIMIT $N\to\infty$

Since the components of the matrix $\hat{T}_1$ are rational functions of the complex variable y, the same should be true also for the components of matrix $\hat{T}_1^N$ and, hence, for the coefficients r_N^a, r_N^s. Representation (15) is useful in that it permits one to employ the identities $|r_N^s|=|r_N^a|=1$, which are valid for any real y. These identities follow from the energy conservation law, since for real y, the nonradiative damping is $\Gamma=0$, and the incoming and outgoing wave energy fluxes coincide in the steady state. The above identities permit recasting $\tilde{r}_N$ in the form

$$\tilde{r}_N = \frac{1}{2}\eta^N\left[\frac{P_{N,s}^{(*)}(y,\eta)}{P_{N,s}(y,\eta)} - \frac{P_{N,a}^{(*)}(y,\eta)}{P_{N,a}(y,\eta)}\right]. \tag{18}$$

Here $P_{N,\gamma}(y,\eta)$ is a polynomial with a unity coefficient of the highest power of y, the other coefficients depending on the single parameter η; the polynomial $P_{N,\gamma}^*(y,\eta)$ is obtained from $P_{N,\gamma}(y,\eta)$ by complex conjugation of all coefficients, including substitution of η^* for η. The common factor η^N in Eq. (18) follows from the limiting recursion relation

$$\lim_{|y|\to\infty} r_N^\gamma = \eta^2 \lim_{|y|\to\infty} r_{N-2}^\gamma$$

and from the behavior of r_0^γ or r_1^γ as $|y|\to\infty$ [see Eq. (17)].

The recursion relation (16) implies a similar connection between the polynomials $P_{N,\gamma}$ and $P_{N-2,\gamma}$, $P_{N-2,\gamma}^*$:

$$P_{N,\gamma}(y) = (y+i)P_{N-2,\gamma}(y) \pm i\eta^{N-1}P_{N-2,\gamma}^{(*)}(y), \tag{19}$$

with the $\pm$ sign referring to the symmetric ($\gamma=s$) or antisymmetric ($\gamma=a$) solutions. For illustration, we present below expressions for the $P_{N,\gamma}(y,\eta)$ polynomials with the few first values of N:

$$P_{1,s}=y+i, \quad P_{1,a}=1,$$

$$P_{2,s}=y+i(1+\eta), \quad P_{2,a}=y+i(1-\eta),$$

$$P_{3,s}=y^2+i(2+\eta^2)y+\eta^2-1, \quad P_{3,a}=y+i(1-\eta^2),$$

$$P_{4,s}=y^2+i(2+\eta+\eta^3)y+\eta^3+\eta^2-\eta-1,$$

$$P_{4,a}=y^2+i(2-\eta-\eta^3)y-\eta^3+\eta^2+\eta-1. \tag{20}$$

The $P_{N,\gamma}(y,\eta)$ polynomials are seen to possess the following properties with respect to the sign reversal of the parameter η:

$$P_{2l+1,\gamma}(y,\eta)=P_{2l+1,\gamma}(y,-\eta),$$

$$P_{2n,s}(y,\eta)=P_{2n,a}(y,-\eta). \tag{21}$$

These properties can be shown by straightforward induction to be valid for arbitrary values of l or n. Equations (18) and (21) yield

$$\tilde{r}_N(y,-\eta)=-\tilde{r}_N(y,\eta), \quad \tilde{t}_N(y,-\eta)=(-1)^N\tilde{t}_N(y,\eta). \tag{22}$$

These expressions relate, in particular, the coefficients $\tilde{r}_N$ and $\tilde{t}_N$ in structures with $kd=\pi(\eta=-1)$ and $kd=2\pi(\eta=1)$.

Another useful expression

$$\tilde{r}_N^*(\omega-\omega_0,\eta)=\tilde{r}_N(\omega_0-\omega,\eta^*),$$

$$\tilde{t}_N^*(\omega-\omega_0,\eta)=\tilde{t}_N(\omega_0-\omega,\eta^*), \tag{23}$$

relates the reflection or transmission coefficients for structures with $kd=\pi+\chi$, i.e. with $\eta=-e^{i\chi}$ and $\eta=-e^{-i\chi}$.

The poles of the coefficient $\tilde{r}_N$ (or, accordingly, the roots of the $P_{N,\gamma}(y)$ polynomials) determine the complex eigenmode frequencies of the heterostructure with due account of the exciton-photon interaction. Note that only even and only odd intrinsic nonstationary excitations contribute, respectively, to r_N^s and r_N^a. As seen from the above $P_{N,\gamma}(y,\eta)$ polynomials, when the number of quantum wells is odd, $N=2l+1$, there are $l+1$ even, and l odd excitations, whereas for $N=2n$ the number of even excitations and of odd excitations are both equal to n. The real and imaginary parts of the roots of the polynomials $P_{N,\gamma}(y)$ determine the eigenfrequency renormalization and the radiative damping of the corresponding mode, i.e. of the exciton polariton, in a system with a finite number of quantum wells. In the general case, all roots of the polynomials $P_{N,s}(y)$ or $P_{N,a}(y)$ are different. A particular case is observed at the cardinal points, $kd=l\pi$ ($l=1,2,...$), i.e. where $\eta=\pm1$. Under this condition all the roots of the polynomials $P_{N,s}(y)$ and $P_{N,a}(y)$ vanish except one. Indeed, for structures with kd equal to a multiple of π we can write the following expressions

$$P_{2l+1,s}(y,\pm1)=y^l[y+i(2l+1)],$$

$$P_{2l+1,a}(y,\pm1)=y^l,$$

$$P_{2n,s}(y,1)=P_{2n,a}(y,-1)=y^{n-1}(y+2ni),$$

$$P_{2n,s}(y,-1)=P_{2n,a}(y,1)=y^n, \tag{24}$$

whose validity can be generally verified by induction, or by setting $\eta=1$ or $\eta=-1$ in Eq. (20). By definition, the polynomials $P_{N,\gamma}^{(*)}(y,\pm1)$ differ from the polynomials (24) by the replacement of iN with $-iN$. When substituting $P_{N,\gamma}(y,-1)$ and $P_{N,\gamma}^{(*)}(y,-1)$ into Eq. (18), the powers of y cancel to yield a simple expression (14a) with a single pole. Thus, as the value of kd approaches π in an N-well structure, $N-1$ eigenmodes do not couple to radiation, leaving only one excitation optically active. This implies that as N increases, exciton interaction with electromagnetic radiation in a structure with $kd=\pi$ decreases. A similar conclusion can be drawn from an analysis of the dispersion relation (7), (8), namely, in a structure with an infinite chain of quantum well structures with $kd=\pi$ the normal wave is the light wave with a wave vector $q=\pi/d$ lying at the bo... the first Brillouin zone, $|\mathrm{Re}\,q|\leqslant\pi/d$. To this value o... responds the solution for the electric field

$$E(z)=E_0\sin(\pi z/d),$$

with the origin of the z axis set at the center of one of th... wells. We consider here the exciton state in a quantum well with a wave function envelope $\Psi(z_e,z_h,\rho)$ even under the replacement of z_e, z_h by $-z_e$, $-z_h$. The light wave (25) does not couple to such an exciton, since the optical transition matrix element proportional to the integral $\int dzE(z)\Psi(z,z,0)$ is zero. Thus under steady-state conditions the light in a resonant Bragg multiquantum well structure with a large number of quantum wells is a structure that does not absorb the standing wave, so that the modulus of the reflection coefficient, $|\tilde{r}_N(kd=\pi)|$, tends to unity as $N\to\infty$ [see Eq. (14a)].

We conclude this Section with an analysis of the limiting expression

$$\tilde{r}_\infty=\lim_{N\to\infty}\tilde{r}_N.$$

We find from Eqs. (11)–(13)

$$\tilde{r}_\infty=\frac{r_1/t_1}{r_1/t_1-F\pm\sqrt{G^2-1}}, \tag{26}$$

where the $\pm$ sign coincides with the sign of $G'=\mathrm{Re}G$. Here the square root of the complex number $z=|z|\exp(i\Theta)$ is defined as $\sqrt{|z|}\exp(i\Theta/2)$, where Θ satisfies the inequalities $-\pi<\Theta\leqslant\pi$. Therefore as $G'\to0$, the function $\sqrt{G^2-1}$ tends to $i\sqrt{1+(G'')^2}\,\mathrm{sign}(G'G'')$, and the quantity $\pm\sqrt{G^2-1}$ in Eq. (26), to $i\sqrt{1+(G'')^2}\,\mathrm{sign}G''$, where $G''=\mathrm{Im}\,G$.

6. RESULTS OF CALCULATIONS

The frequency dependence of the reflection coefficient can be conveniently presented as the trajectory the point $\mathrm{Re}\,\tilde{r}_N$, $\mathrm{Im}\,\tilde{r}_N$ makes on the $\tilde{r}_N$ complex plane as the point ω moves along the real axis [Fig. 2a]. Since the coefficient $\tilde{r}_N\to0$ with increasing $|\omega-\omega_0|$, these trajectories are closed curves leaving and coming back to point $\tilde{r}_N=0$. For $N=1$, the $\tilde{r}_1(\omega)$ dependence may be considered as a bilinear transformation which transforms the real axis into a circle passing through the center of the $\tilde{r}_N$ complex plane. As seen from Eq. (14a), the trajectories corresponding to arbitrary N in structures with $kd=\pi$, 2π,... behave in the same way. For $N>1$, $kd\neq1\pi$, and $\Gamma/\Gamma_0\leqslant1$ these trajectories have a complicated shape including self-crossings. As the nonradiative damping increases, the self-crossings disappear, and the trajectory contracts toward the center and approaches a circular shape.

Figure 2b displays the reflectance spectrum calculated using Eq. (1) for a structure with $N=5$, $\Gamma=\Gamma_0$, $kd=\pi/4$, and $kl=\pi/2$, where l is the width of the barrier layer between the outer surface and the plane separated by $d/2$ from the left-hand-well center (see Fig. 1). The calculation took into account that at normal incidence

$$r_{10}=-r_{01}=\frac{\sqrt{\varepsilon_b}-1}{\sqrt{\varepsilon_b}+1}. \tag{27}$$

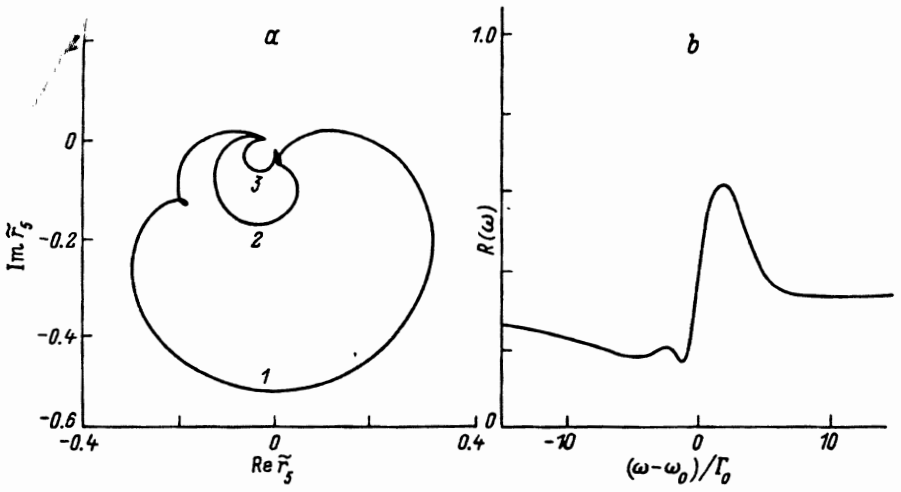

FIG. 2. (a) Trajectories traced out by the point $\mathrm{Re}\,\tilde{r}_5$, $\mathrm{Im}\,\tilde{r}_5$ on the complex plane as the frequency ω moves along the real axis, for $kd=\pi/4$ structures with $\Gamma/\Gamma_0=1$ (curve 1), 4(curve 2), and 12 (curve 3). (b) Normal-incidence reflectance spectrum calculated using Eq. (1) for a structure with $N=5$, $kd=\pi/4$, $kl=\pi/2$, and $\Gamma=\Gamma_0$, and barrier layer refractive index $\sqrt{\varepsilon_b}=3.7$.

The second maximum in Fig. 2b is due to a self-crossing of the corresponding trajectory in Fig. 2(a).

Equations (1) and (14a) yield for a Bragg structure

$$R(\omega)=r_{01}^2+(1-r_{01}^2)\frac{N\Gamma_0(N\Gamma_0+\Omega)}{(\omega_0-\omega)^2+(\Gamma+N\Gamma_0)^2+N\Gamma_0\Omega}, \quad (28)$$

where

$$\Omega=r_{01}^2N\Gamma_0+2r_{01}[(\omega_0-\omega)\sin 2\phi' +(\Gamma+N\Gamma_0)\cos 2\phi'].$$

At $\phi'=\pi/2$ the $R(\omega)$ spectrum is a sum of the background reflectance r_{01}^2 and a Lorentzian

$$L(\omega)=\rho\frac{\gamma^2}{(\omega_0-\omega)^2+\gamma^2},$$

where

$$\gamma=\Gamma+(1-r_{01})N\Gamma_0,$$

$$\rho=(1-r_{01}^2)_s\frac{s(1+r_{01}^2)-2r_{01}}{(1-r_{01}s)^2}, \quad (29)$$

and $s=N\Gamma_0/(\Gamma+N\Gamma_0)$. The spectrum retains its shape on approaching the values $\phi'=0$ or π, while the damping parameter γ and the peak value of ρ are given by Eqs. (29), where the sign of r_{01} should be reversed. This results in a decrease of the integral $\int d\omega[R(\omega)-r_{01}^2]=\pi\gamma\rho$.

A calculation of the reflectance spectra made for $\Gamma/\Gamma_0\leq 1$ and various values of N, kd, and kl shows them to exhibit a highly diverse fine structure, despite the fact that the spectral region covered includes one of the exciton resonances. Figure 3 displays spectra computed for a structure with $N=10$, $\phi'\equiv kl=\pi/2$, and kd equal to 0.85π, π, and 1.15π. The smooth spectrum for the structure with $kd=\pi$ is described by a simple expression $r_{01}^2+L(\omega)$. The spectra for $kd=\pi(1\pm 0.15)$ each have two maxima and one minimum. They are seen to transform into one another under reflection about the vertical axis $\omega=\omega_0$. This property is a consequence of Eq. (23), which relates the coefficients $\tilde{r}_N$ corresponding to the complex conjugate values of η, and the fact that the phase factor $e^{i2\phi'}$ is real at $\phi'=\pi/2$.

7. OBLIQUE REFLECTION GEOMETRY

Let us introduce an angle φ at which light is incident from vacuum onto a structure. The relations derived above for the normal-incidence reflection coefficient become valid for an s-polarized light wave incident at $\varphi=0$ after the replacement

$$kd\to k_zd,\quad kl\to k_zl,\quad \Gamma_0\to\frac{k}{k_z}\Gamma_0,$$

$$r_{01}=-r_{10}=-\frac{n_z-\cos\varphi}{n_z+\cos\varphi}.$$

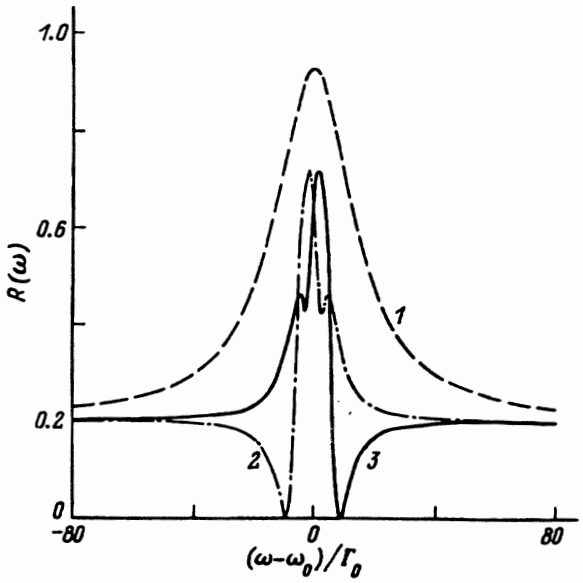

FIG. 3. Normal-incidence reflectance spectra calculated for a structure with $N=10$, $kl=\pi/2$, $\Gamma=\Gamma_0$, $kd=\pi$ (curve 1), 0.85π (curve 2), and 1.15π (curve 3), and barrier layer refractive index $\sqrt{\varepsilon_b}=2.64$.

Here

$$n_z = ck_z/\omega, \quad k_z = (\omega/c)(\varepsilon_b - \sin^2 \varphi)^{1/2}.$$

The expressions for r_1 obtained for a p polarized wave in the general case where the oscillator strength of the exciton under study is nonzero for $\mathbf{E} \perp z$ and $\mathbf{E} \| z$ (z is the principal axis of the structure) are more cumbersome.[13] However in the case of the heavy exciton in GaAs/AlGaAs or CdTe/CdMgTe structures, which is optically active only in the $\mathbf{E} \perp z$ polarization, the relations derived by us earlier can be readily generalized by the replacement

$$kd \to k_z d, \quad kl \to k_z l, \quad \Gamma_0 \to \frac{k_z}{k} \Gamma_0,$$

$$r_{01} = \frac{n_z - \varepsilon_b \cos \varphi}{n_z + \varepsilon_b \cos \varphi}.$$

In place of studying the fine structure evolution of optical spectra under a smooth variation of the period d one can instead measure the angular dependence of these spectra.

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