

# Мезоскопический неэрмитовый скин-эффект

[А.Н. Поддубный](#)

collaboration: [Janet Zhong](#) and [Shanhui Fan](#) (Stanford)



**Weizmann Institute of Science, Israel**

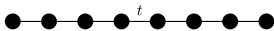
[weizmann.ac.il/complex/poddubny](http://weizmann.ac.il/complex/poddubny)

[poddubny@weizmann.ac.il](mailto:poddubny@weizmann.ac.il)

Низкоразмерный семинар, 15 января 2024, онлайн

# Periodic vs open boundary conditions (PBC vs OBC)

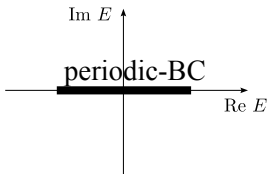
Model:



$$\varepsilon\psi_n = t(\psi_{n+1} + \psi_{n-1})$$

$$t = t^*$$

Energy spectrum:

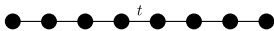


$$E(k) = te^{ik} + te^{-ik}$$

$$\psi_n \propto e^{ikn} \text{ (periodic)}$$

# Periodic vs open boundary conditions (PBC vs OBC)

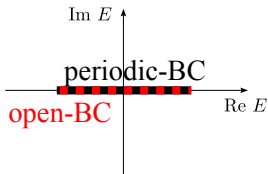
Model:



$$\varepsilon\psi_n = t(\psi_{n+1} + \psi_{n-1})$$

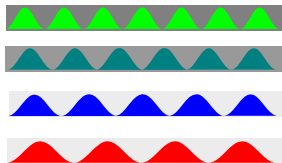
$$t = t^*$$

Energy spectrum:



$$E(k) = te^{ik} + te^{-ik}$$

Eigenmodes:



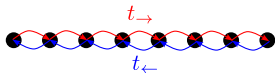
$$\psi_n \propto e^{ikn} \text{ (periodic)}$$

$$\psi_n \propto e^{ikn} + re^{-ikn}$$

(open BC)

# PBC vs OBC in non-Hermitian structures

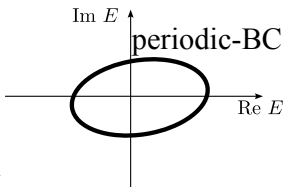
Model:



$$\varepsilon\psi_n = t_{\leftarrow}\psi_{n+1} + t_{\rightarrow}\psi_{n-1}$$

N. Hatano and D. R. Nelson, PRB  
56, 8651 (1997)

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$$E(k) = t_{\leftarrow}e^{ik} + t_{\rightarrow}e^{-ik}$$

$$\psi_n \propto e^{ikn} \text{ (periodic)}$$

T.E. Lee, PRL 116, 133903 (2016)

D. Leykam, K. Y. Bliokh, C. Huang, Y. D. Chong, and F. Nori, PRL 118, 40401 (2017)

V. M. Martinez Alvarez, J. E. Barrios Vargas, and L. E. F. Foa Torres PRB 97, 121401(R) (2018)

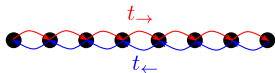
Reviews: Bergholtz, Budich, Kunst, Rev. Mod. Phys. 93, 015005 (2021)

Nobuyuki Okuma, and Masatoshi Sato, Annu. Rev. Cond. Mat. Phys. 14, 83 (2023)

Q. Wang and Y. D. Chong, JOSA B 40, 1443 (2023)

# PBC vs OBC in non-Hermitian structures

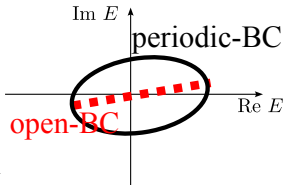
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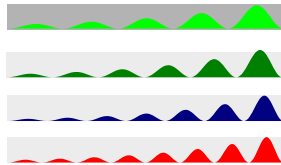
N. Hatano and D. R. Nelson, PRB  
56, 8651 (1997)

Energy spectrum:



$$E(k) = t_{\leftarrow}e^{ik} + t_{\rightarrow}e^{-ik}$$

Eigenmodes:



$$\begin{aligned} \psi_n &\propto e^{ikn} \text{ (periodic)} \\ \psi_n &\propto e^{ik_+n} + r e^{ik_-n} \\ &\text{(open BC)} \\ \text{Im } k_+ &+ \text{Im } k_- > 0 \end{aligned}$$

## non-Hermitian skin effect

- complex  $E(k)$  under PBC has non-zero winding number
- complex  $E$  under OBC collapses into a line
- OBC modes are concentrated at the edge

T.E. Lee, PRL 116, 133903 (2016)

D. Leykam, K. Y. Bliokh, C. Huang, Y. D. Chong, and F. Nori, PRL 118, 40401 (2017)

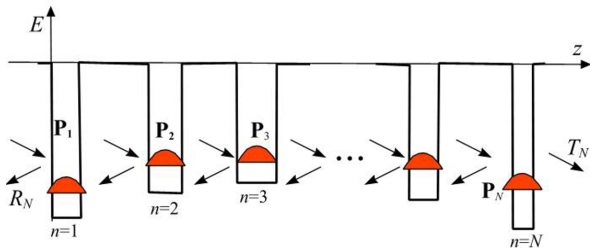
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Nobuyuki Okuma, and Masatoshi Sato, Annu. Rev. Cond. Mat. Phys. 14, 83 (2023)

Q. Wang and Y. D. Chong, JOSA B 40, 1443 (2023)

## Модель распространения экситонных поляритонов в структуре



$$x = \Gamma/\Gamma_0$$

$$P_{\text{exc}}(z) = \sum_{n=1}^N P_n a \delta(z - z_n) = \xi \sum_{n=1}^N \frac{\Gamma_0}{\omega_n - \omega - i\Gamma} \delta(z - z_n) E(z_n)$$

$$P_n = \frac{1}{a} \int_{(n)} dz P_{\text{exc}}(z) \quad \xi = \frac{\varepsilon_b}{2\pi k_b a}$$

$$(\omega_n - \omega - i\Gamma)P_n - i\Gamma_0 \sum_{n'=1}^N P_{n'} e^{ik_b |z_n - z_{n'}|} = \xi \Gamma_0 E_0(z_n)$$

Слайд – АНП и В.А. Кособукин, Экситон-поляритонное поглощение в квантовых ямах: от короткопериодных структур к брэгговским, Низкоразмерный семинар, 2007

# Coupled quantum wells: eigenfrequencies

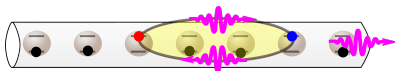
Физика и техника полупроводников, 1998, том 32, № 1

## Экситонные поляритоны в длиннопериодных структурах с квантовыми ямами

© М.Р. Владимирова, Е.Л. Ивченко, А.В. Кавокин

Физико-технический институт им. А.Ф.Иоффе Российской академии наук,  
194021 Санкт-Петербург, Россия

(Получена 29 мая 1997 г. Принята к печати 3 июня 1997 г.)



## Уравнение для собственных частот в методе связанных осцилляторов

В этом методе собственные частоты находятся из уравнения

$$\det \| A_{lm} - \omega \delta_{lm} \| = 0,$$
$$A_{lm} = (\omega_0 - i\Gamma) \delta_{lm} - i\Gamma_0 e^{ik|l-m|}, \quad (11)$$

которое полностью эквивалентно уравнению (5). Так как след любой квадратной матрицы равен сумме ее

# Coupled quantum wells: eigenfrequencies

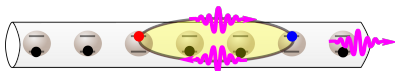
Физика и техника полупроводников, 1998, том 32, № 1

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© М.Р. Владимирова, Е.Л. Ивченко, А.В. Кавокин

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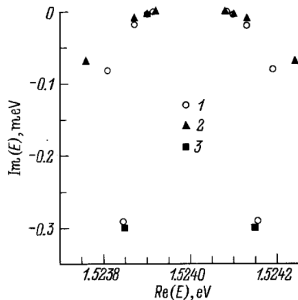


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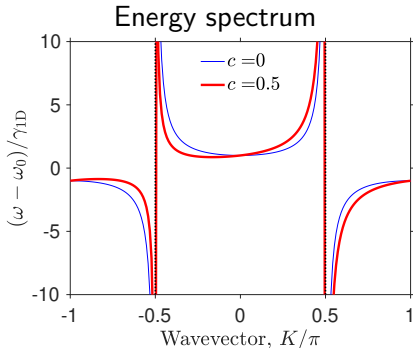
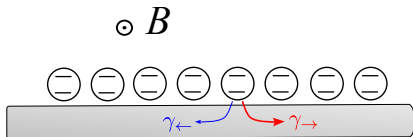
которое полностью эквивалентно уравнению (5). Так как след любой квадратной матрицы равен сумме ее



**Рис. 3.** Комплексные собственные частоты для экситонных поляритонов в антибрезгтовской структуре ( $d = d_A$ ) с 10 квантовыми ямами. 1 — точный расчет, представленный также на рис. 1, b, 2 — расчет по формулам (14), (17), 3 — расчет по приближенной формуле (18).



# Chiral waveguide quantum electrodynamics



Markovian approx., single exc.:

$$H_{m,n} = -i \begin{cases} \gamma_{\rightarrow} e^{i\varphi|m-n|}, & m > n \\ \frac{\gamma_{\rightarrow} + \gamma_{\leftarrow}}{2}, & m = n, \\ \gamma_{\leftarrow} e^{i\varphi|m-n|}, & m < n \end{cases} \quad \omega(K) = \gamma_{1D} \frac{\sin \varphi + c \sin K}{\cos K - \cos \varphi},$$

$$c = \frac{1-\xi}{1+\xi}, \quad \xi = \gamma_{\rightarrow}/\gamma_{\leftarrow}.$$

Review: P. Lodahl, S. Mahmoodian, S. Stobbe, A. Rauschenbeutel, P. Schneeweiss, J. Volz, H. Pichler, and P. Zoller, *Nature* **541**, 473 (2017).

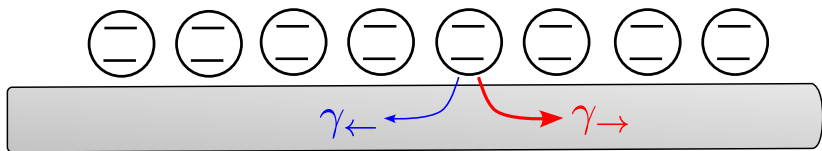
D. F. Kornovan, M.I. Petrov, and I. V. Iorsh, *PRB* **96**, 115162 (2017), семинар от 27.02.2017  
G. Fedorovich, D. Kornovan, ANP, and M. Petrov, *PRA* **106**, 043723 (2022)

More on waveguide QED: A.S. Sheremet, M.I. Petrov, I.V. Iorsh, A.V. Poshakinskiy and ANP, *Rev. Mod. Phys.* **95**, 015002 (2023);

видео, cond-mat.ru: <https://www.youtube.com/live/6zewVeRcG-Y?si=LazlQQbFWsbKC54X>

# Effective permittivity for short-period structure

$\odot B$

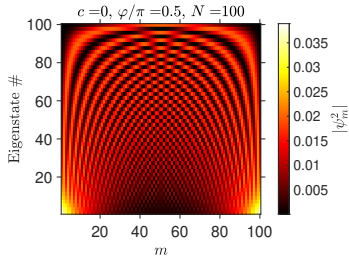
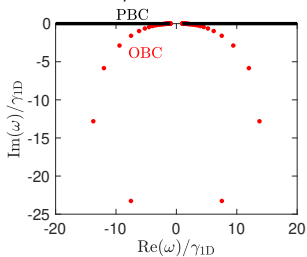


$$\varepsilon_{\text{eff}}(\omega, K) = 1 + \frac{2\gamma_{1D}}{\varphi} \frac{1 + cK}{\omega_0 - \omega}$$

Non-chiral case,  $c = 0$ : [Е.Л. Ивченко, ФТТ 33, 2388\(1991\)](#)

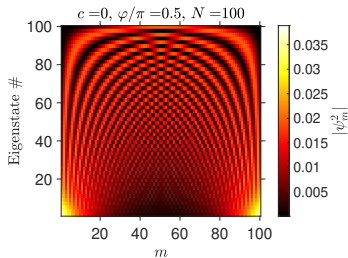
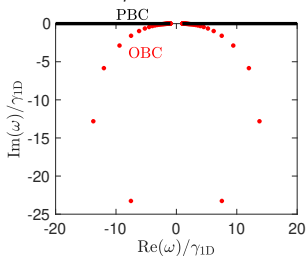
# chiral WQED vs non-chiral WQED

Nonchiral,  $c = 0$

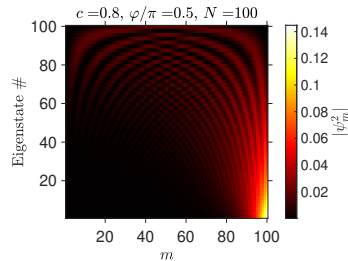
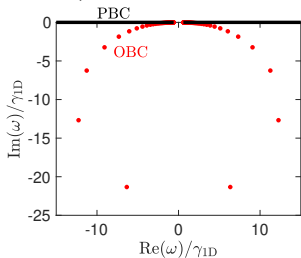


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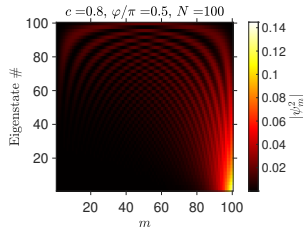
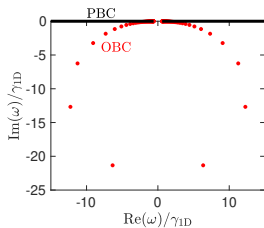
Chiral,  $c = 0.8$



# chiral WQED vs Hatano-Nelson



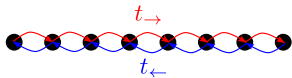
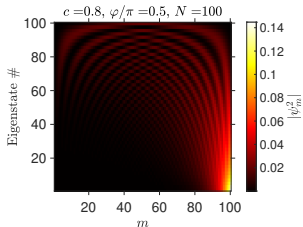
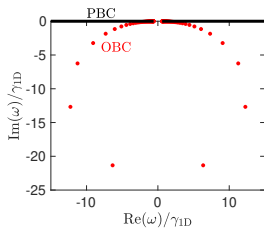
$$\frac{\omega(k)}{\gamma_{1D}} = \frac{\sin \varphi + c \sin k}{\cos k - \cos \varphi},$$



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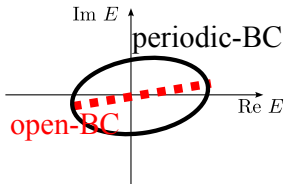


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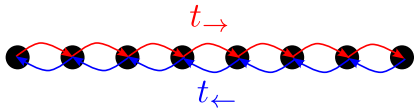


$$E(k) = t_{\leftarrow} e^{ik} + t_{\rightarrow} e^{-ik}$$

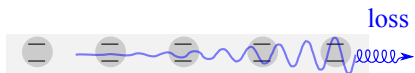
N. Hatano and D. R. Nelson, PRB  
56, 8651 (1997)



# What does “periodic non-Hermitian structure” mean?



Distributed loss:  $\text{rank Im } H \propto \text{size}$



Edge loss:

$\text{rank Im } H = \text{const}, \text{rank Im } H / \text{rank Re } H \rightarrow 0$

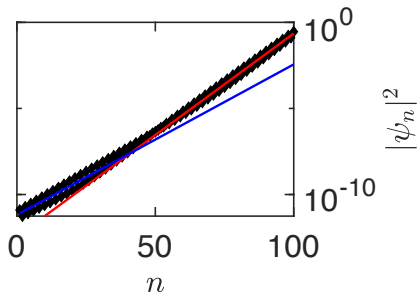
# Mesoscopic vs conventional NHSE

$$\psi_n \propto e^{iK_+n} + re^{iK_-n}, \quad n = 1 \dots N$$

Conventional:

$$\text{Im } K_+ \text{Im } K_- > 0$$

$$K_+, K_- = \text{const}(N)$$





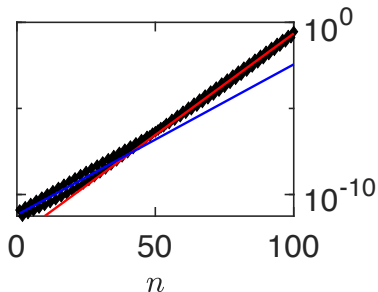
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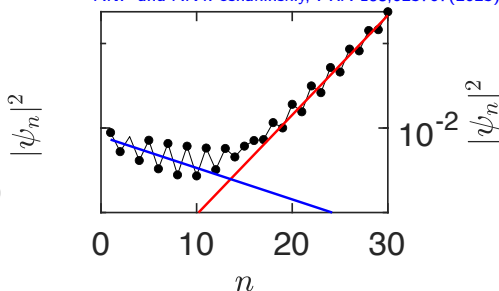
Mesoscopic:

$$\text{Im } K_+ \text{Im } K_- < 0$$

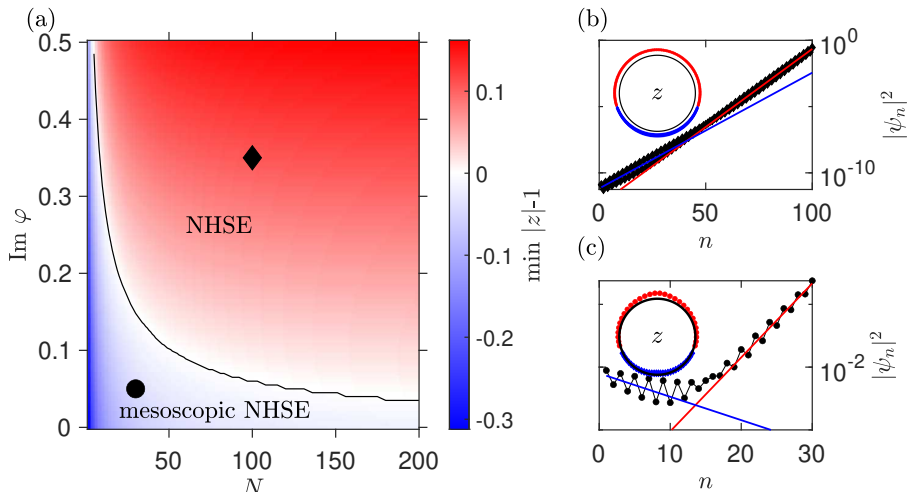
$$K_+(N), K_-(N),$$

$$|\psi_{N/2}|^2 / |\psi_1|^2 \propto 1/N$$

ANP and A.V.Poshakinskiy, PRA 108,023707(2023)



# Transition to normal NHSE with losses



$$H_{mn} \propto e^{i\varphi|m-n|}$$

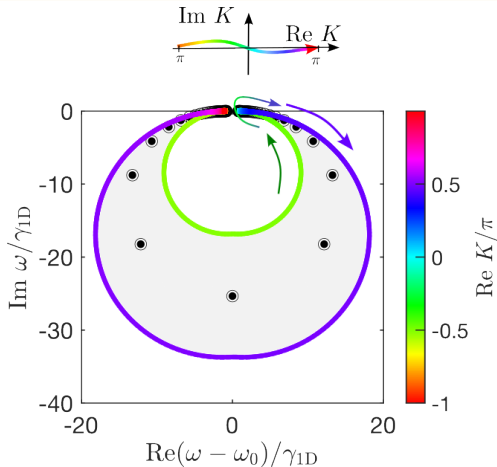
$$z_{\pm} = \exp(iK_{\pm}), \psi_m \propto z_+^m + rz_-^m, \quad m = 1 \dots N$$

# Regularized PBC spectrum and mesoscopic NHSE

$$k \rightarrow \text{Re } k - i\delta(N) \sin(\text{Re } k)$$

$$-\pi < \text{Re } k \leq \pi$$

$$\frac{\omega(k)}{\gamma_{1D}} = \frac{\sin \varphi + c \sin k}{\cos k - \cos \varphi},$$

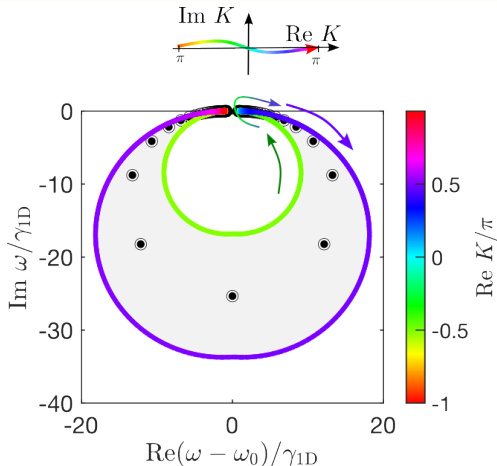


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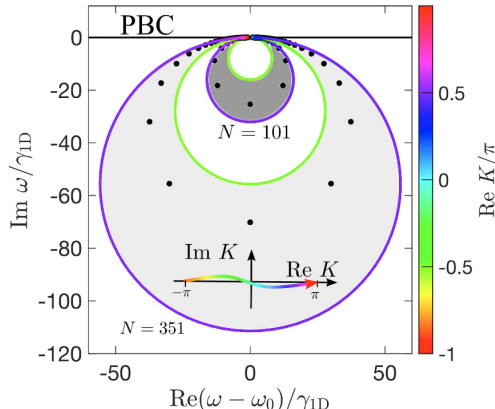
$$\text{analytics for OBC: } \omega_{\nu}^{\pm} = -\frac{iN\gamma_{1D}}{W_{\nu} \left( \pm 2N/\sqrt{1-c^2} \right)}, \quad \nu = 0, \pm 1, \pm 2 \dots$$

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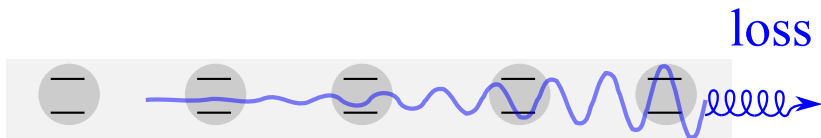


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analytics for  $c = 0$ : [A.V. Poshakinskiy and ANP, PRA 108, 023707 \(2023\)](#)

## Summary: mesoscopic NHSE

- The concept of NHSE potentially applies also to finite-size photonic structures with neither gain nor loss in the bulk and purely real energy spectrum under periodic boundary conditions (PBC)
- regularized size-dependent PBC spectrum
- definition of non-Hermitian periodic system requires clarification



[arXiv:2310.04025v1](https://arxiv.org/abs/2310.04025v1)



## Some equations

Dispersion law:  $w(K) = \frac{\sin \varphi + \chi \sin K}{\cos K - \cos \varphi}$ , where  $\chi = \frac{1 - \xi}{1 + \xi}$ .

Eq. for spectrum:  $r_{\rightarrow}(\omega)r_{\leftarrow}(\omega)e^{i(K_+ - K_-)(N-1)} = 1$ ,  $r_{\rightleftarrows} = -\frac{e^{\pm iK_{\pm}} - e^{i\varphi}}{e^{\pm iK_{\mp}} - e^{i\varphi}}$

Eigenstates:  $\psi_m \propto e^{iK_+(m-N)} + r_{\leftarrow} e^{iK_-(m-N)} \propto e^{iK_+(m-1)} r_{\rightarrow} + e^{iK_-(m-1)}$ ,

$$K_{\pm} \approx \pm \frac{\pi}{2} + (\chi \mp 1) \frac{\gamma_{1D}}{\omega - \omega_0}.$$

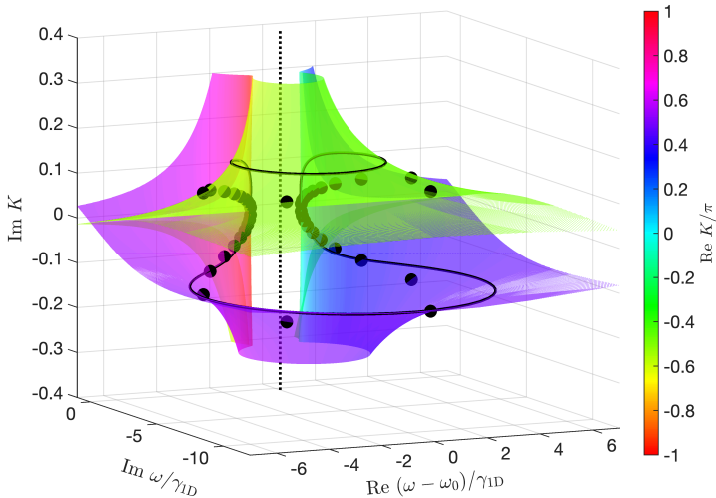
$$r_{\rightarrow} \approx -\frac{i(\chi + 1)}{2\omega}, \quad r_{\leftarrow} \approx -\frac{i(1 - \chi)}{2\omega}, \quad r_{\leftarrow} r_{\rightarrow} e^{i(K_+ - K_-)} \approx \frac{1 - \chi^2}{4\omega^2}.$$

$$\frac{1 - \chi^2}{4\omega^2} e^{-\frac{2iN}{\omega}} + 1 = 0.$$

Spectrum:  $\omega_{\nu}^{\pm} - \omega_0 = -\frac{iN\gamma_{1D}}{W_{\nu} \left( \pm 2N / \sqrt{1 - \chi^2} \right)}, \quad \nu = 0, \pm 1, \dots$



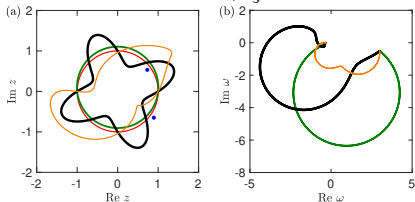
# Riemann surface of $\omega(k)$



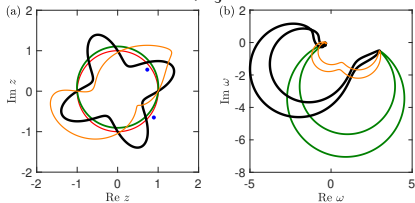
# More topological arguments

$$w(K) = \frac{\sin \varphi + \chi \sin K}{\cos K - \cos \varphi}, \quad z = e^{iK}$$

Non-chiral,  $\xi = 0$



Chiral,  $\xi = 0.8$



## Loss/chirality phase diagram

$$H_{m,n} = -i \begin{cases} \gamma_{\rightarrow} e^{i\varphi|m-n|}, & m > n \\ \frac{\gamma_{\rightarrow} + \gamma_{\leftarrow}}{2}, & m = n \\ \gamma_{\leftarrow} e^{i\varphi|m-n|}, & m < n \end{cases},$$

$$H_{nm} - H_{mn}^* = -i\gamma_{\rightarrow} e^{i\varphi(n-m)} - i\gamma_{\leftarrow} e^{i\varphi(m-n)}$$

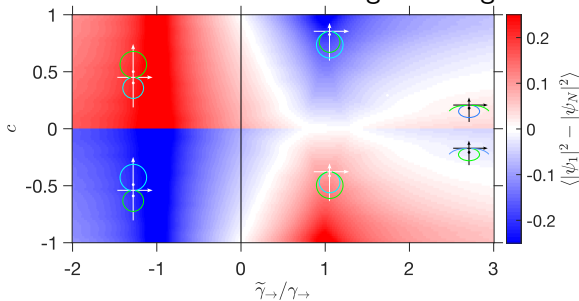
$$H_{nm} - H_{mn}^* = -i\tilde{\gamma}_{\rightarrow} e^{i\varphi(n-m)} - i\gamma_{\leftarrow} e^{i\varphi(m-n)}$$

# Loss/chirality phase diagram

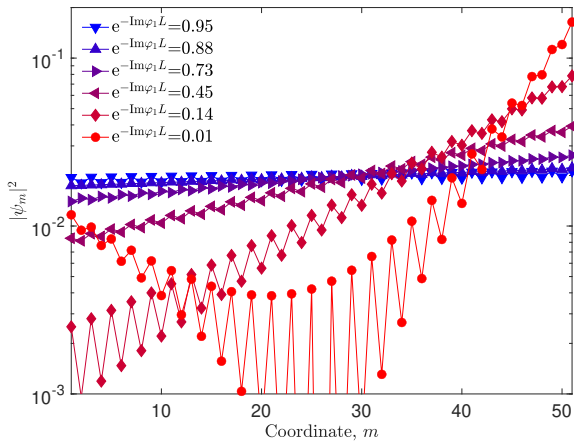
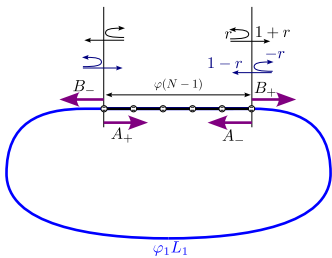
$$H_{m,n} = -i \begin{cases} \gamma_{\rightarrow} e^{i\varphi|m-n|}, & m > n \\ \frac{\gamma_{\rightarrow} + \gamma_{\leftarrow}}{2}, & m = n \\ \gamma_{\leftarrow} e^{i\varphi|m-n|}, & m < n \end{cases},$$

$$H_{nm} - H_{mn}^* = -i\tilde{\gamma}_{\rightarrow} e^{i\varphi(n-m)} - i\gamma_{\leftarrow} e^{i\varphi(m-n)}$$

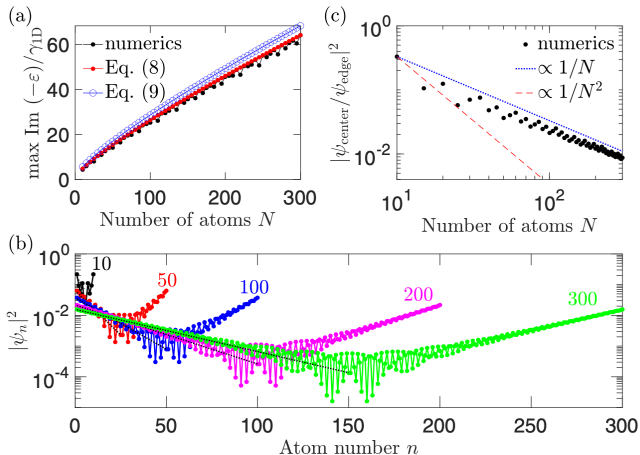
Let us tune losses through the edges.



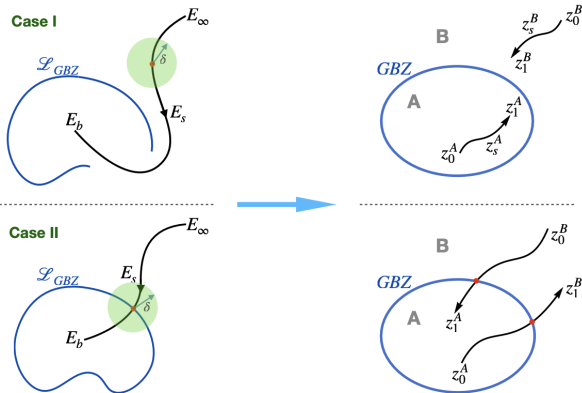
# Circular waveguide



# Eigenstate localization in a non-chiral structure



# Topological arguments



$$H(z) = t_0 + t_1 z + \frac{t_{-1}}{z} + \dots, \quad z = e^{ik}$$

$$GBZ \equiv \{z, |H_m[H(z)]| = |H_{m+1}^{-1}[H(z)]|\}$$

$$\text{winding}(GBZ) = \frac{1}{2\pi} \oint dz \arg [H(z) - E_b] = N_{\text{zeroes}} - N_{\text{poles}} = 0$$

# Some more literature

- M. Yang, L. Wang, X. Wu, H. Xiao, D. Yu, L. Yuan, and X. Chen, PRA **106**, 043717 (2022)  
PBC is complex; inherent **radiative** losses perpendicular to the array plane,
- Yi-Cheng Wang, Jih-Shih You, and H. H. Jen, Nat. Commun **13**, 4598 (2022)  
PBC is complex; inherent **radiative** losses perpendicular to the array plane
- Tao Yu and Zeng, PRB **105**, L180401 (2022); PRR **5**, 013003 (2023):  
PBC is complex; inherent **nonradiative** losses

