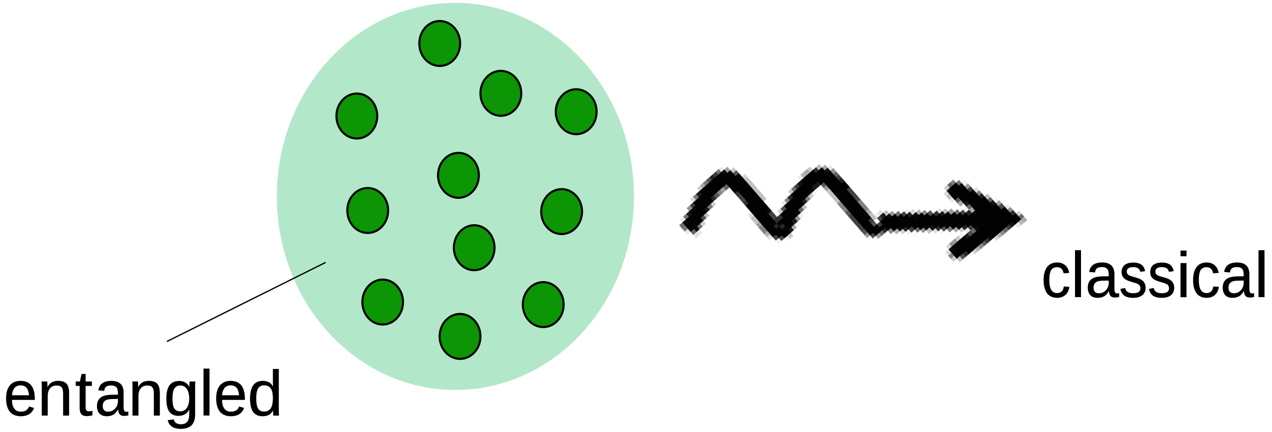
Quantum entanglement at the origin of classical radiation



microscopic quantum entanglement *essential for* establishing **macroscopic classical** response

AMOS retreat, Neve Ilan, May 2022

מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

הקרן הלאומית למדע
المؤسسة الإسرائيلية للعلوم
Israel Science Foundation

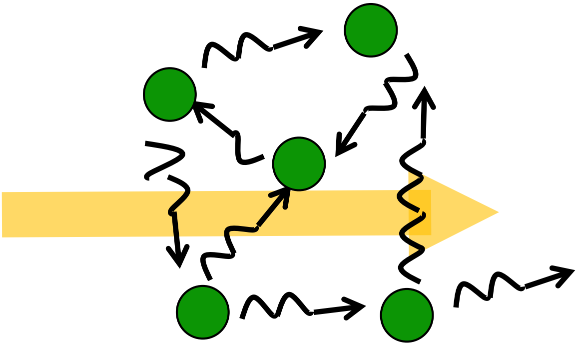
Efi Shahmoon

Weizmann Institute of Science, Israel

Collective radiation: Superradiance

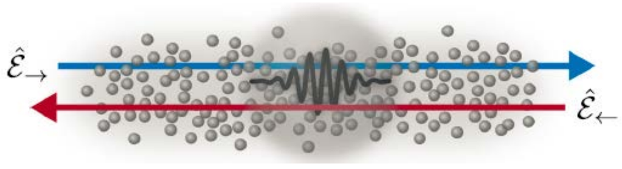
Radiation from dense ensemble of emitters (= "atoms")

Collective = multiple photon scattering (dipole-dipole) significant



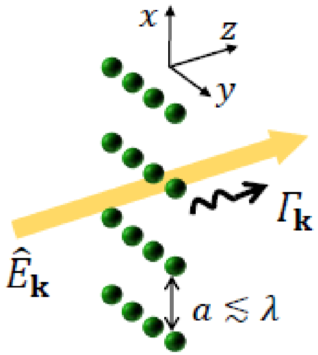
Relevant in many systems & applications

atomic ensembles



credit: Murray & Pohl (2017)

optical lattice, clocks



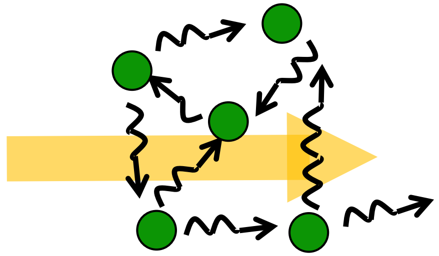
light harvesting complexes

photonics

In general: **unsolved, fundamental** many-body problem

- nonlinear, open system, non-equilibrium

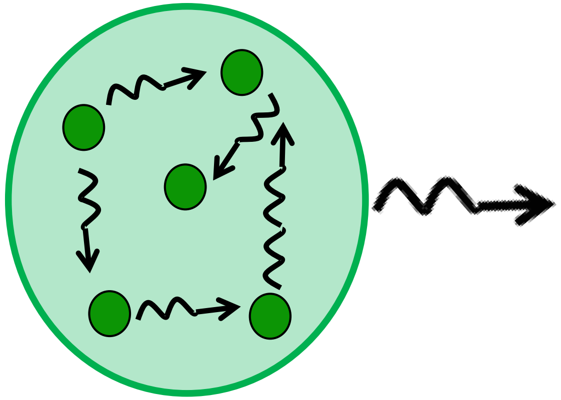
Collective radiation: Superradiance



"canonical" case: **Dicke superradiance**

all atoms identically coupled to field (at same "point"): permutation symmetry

→ many atoms = one "giant" spin (**macroscopic dipole**)

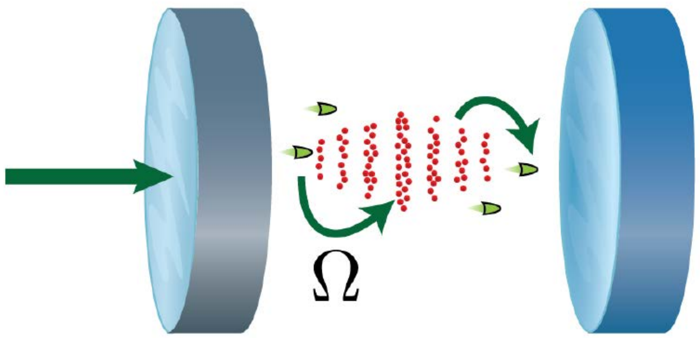


$$N \times \left(\begin{matrix} \hat{\sigma}_n = |g\rangle_n \langle e| \\ \text{1 atom: spin } 1/2 \end{matrix} \right) \Rightarrow \hat{J} = \sum_{n=1}^{N} \hat{\sigma}_n$$

spin-j: $j = N/2$

[direct product basis, 2^N states] → [direct sum basis, symmetric j=N/2 subspace]

Realization: cavity/waveguide QED



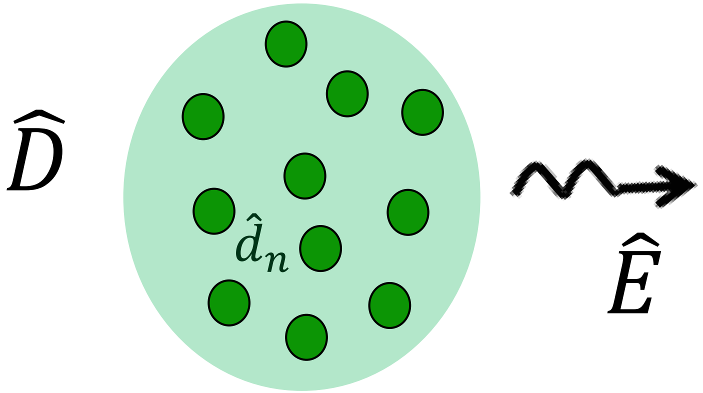
spin-j: basis states $|j, m\rangle$ with $m \in \{-j, j\}$

$2j + 1 = N + 1$ states

→ symmetric "Dicke" states

Macroscopic dipole → classical limit?

Macroscopic: $N \gg 1$ constituents $\hat{D} = \sum_{n=1}^{N} \hat{d}_n$



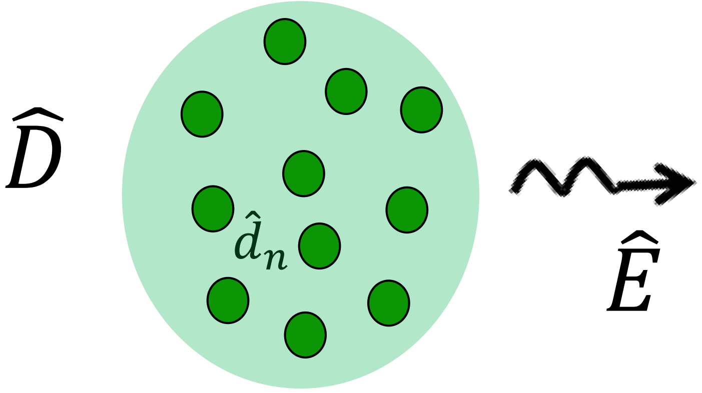
$N \gg 1$

Question: Is there a classical antenna-radiation limit?

“classical” radiation: Coherent state $\hat{E}|\psi\rangle \propto |\psi\rangle$ of the field

What makes a dipole radiate classical-like coherent state?

Macroscopic dipole → classical limit?



Question: what makes a dipole radiate "classical" coherent state? $\hat{E}|\psi\rangle \propto |\psi\rangle$

→ generic dipole-field coupling (RWA) $\hat{H} = \hat{D}^\dagger \hat{E} + \hat{E}^\dagger \hat{D}$

Answer: when it is pumped to a dipole eigenstate $\hat{D}|\psi(t)\rangle = \alpha|\psi(t)\rangle$

Explanation: effectively (for field), $\hat{H}_{\mathrm{eff}} = \alpha^* \hat{E} + \alpha \hat{E}^\dagger \quad \rightarrow \quad \hat{U} = e^{-i\hat{H}_{\mathrm{eff}} t} = e^{i\alpha t \hat{E}^\dagger - i\alpha^* t \hat{E}}$

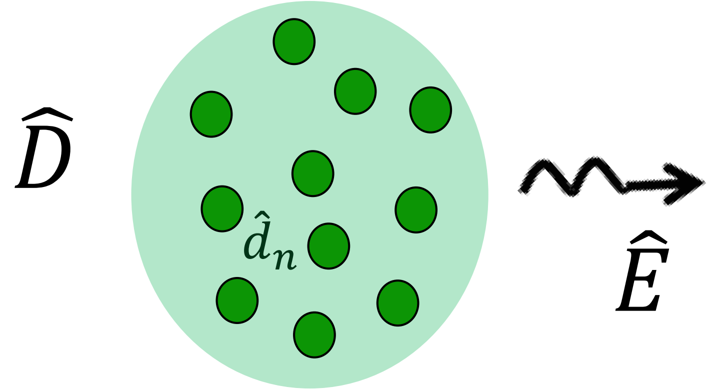
→ generates a coherent-state field $\hat{E}|\psi\rangle = i\,\alpha\, t|\psi\rangle$

Example 1: linear system: $\hat{D}$ Harmonic-oscillator lowering operator

Classical state of the dipole ⟷ Classical state of the field

Quantum-Classical "correspondence"

Macroscopic dipole → classical limit?



generic dipole-field coupling (RWA) $\hat{H} = \hat{D}^\dagger \hat{E} + \hat{E}^\dagger \hat{D}$

Question: how does a dipole radiate "classical" coherent state? $\hat{E}|\psi\rangle \propto |\psi\rangle$

Answer: when it is pumped to a dipole eigenstate $\hat{D}|\psi(t)\rangle = \alpha|\psi(t)\rangle$

Example 2: nonlinear system, spin $j = N/2$ $\hat{D} = \hat{J}$ SU(2) lowering operator

Q1: Do eigenstates of $\hat{J}$ exist?

A1: Yes! For $N = 2j \gg 1$

Q2: is there Q-C correspondence?

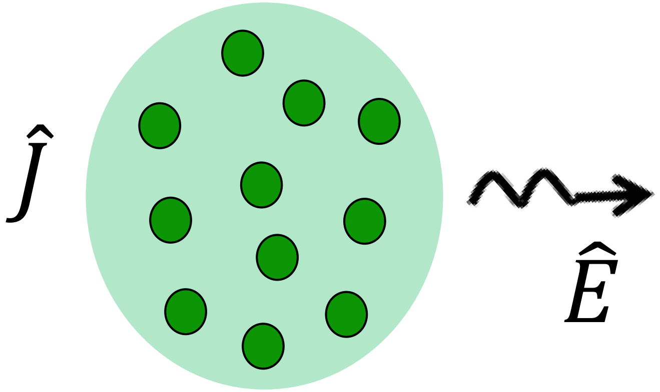
A2: not really! They are entangled states

Q-entangled state of the dipole (macro-spin) → Classical state of the field

Coherently radiating spin states: "CRSS"

Outline

CRSS = coherently radiating spin states $\hat{J}|\alpha\rangle = \alpha|\alpha\rangle$



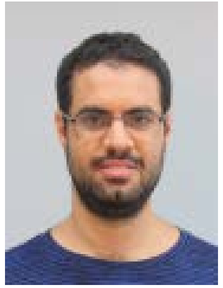
1. CRSS exist: eigenstates of $\hat{J}$

2. CRSS are physical: underlie steady-state of superradiance

3. CRSS are entangled: spin squeezing $N^{-1/3}$

4. CRSS radiate classically: dipole-projected squeezing

5. outlook



Ori Somech
(MSc student)

CRSS exist: Asymptotic eigenstates of $\hat{J}$

Look for eigenstates:

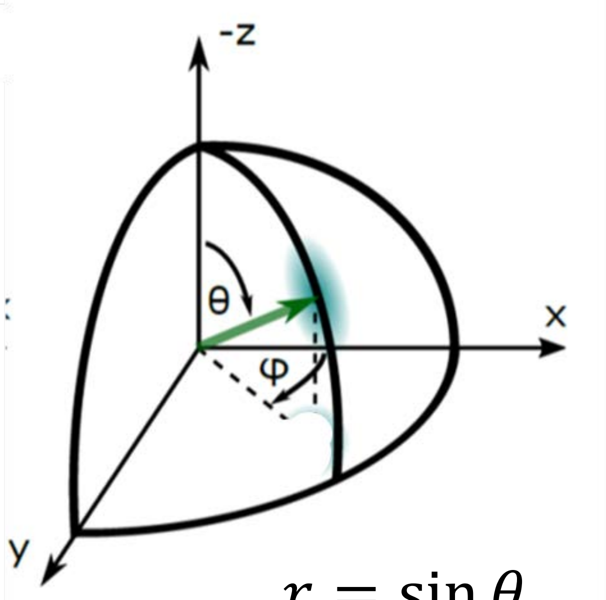
$$\hat{J}|\alpha\rangle = \alpha|\alpha\rangle$$

$\hat{J} =$ Spin-j lowering operator

$\alpha = jre^{i\varphi} =$ Complex amplitude (eigenvalue)

In general: no eigenstates apart from $|j,-j\rangle = |\alpha = 0\rangle$

→ We find approximate eigenstates for $|\alpha| < j$ in the limit $j \to \infty$



$r = \sin\theta$

Formally:

Define proximity error $\epsilon = \left\| \hat{J}|\alpha\rangle - \alpha|\alpha\rangle \right\|$

Demand: $\lim_{j\to\infty} \epsilon = 0$ keeping $r = |\alpha|/j$ fixed

while taking the limit $j \to \infty$

- We find a state that minimizes the error

- Show error vanishes for $j \to \infty$ → **CRSS**

CRSS exist: Asymptotic eigenstates of $\hat{J}$

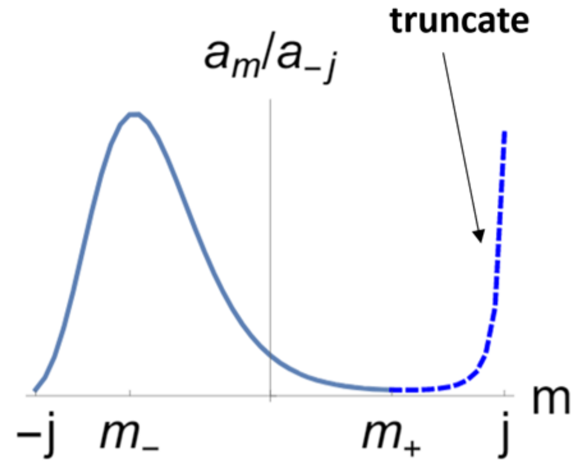
Look for eigenstates:

$$(1) \quad \hat{J}|\alpha\rangle = \alpha|\alpha\rangle$$

$\hat{J} =$ Spin-j lowering operator

$\alpha = jre^{i\varphi} =$ Complex amplitude (eigenvalue)

Insert a general state into Eq. (1) $|\psi\rangle = \sum_{m=-j}^{s} a_m |j, m\rangle \qquad s \leq j$



$$\alpha^2 = j(j+1) - m(m+1)$$

roots ⟶ $m_\pm$

→ obtain:

1. recursion relations for coeff. $a_{m+1} = \dfrac{\alpha}{\sqrt{j(j+1) - m(m+1)}} a_m$

2. “inconsistent” result: $\epsilon = \left\| \hat{J}|\psi\rangle - \alpha|\psi\rangle \right\| = j|a_s| \neq 0$

→ Minimize error:

Find s for which $|a_s| = \epsilon/j$ minimal

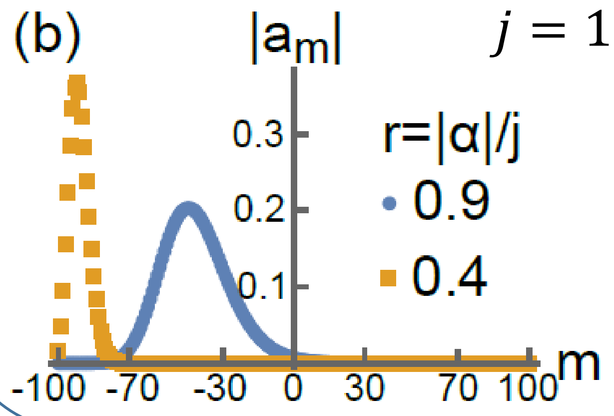
→ Our CRSS ansatz: $|\alpha\rangle = \sum_{m=-j}^{m_+} a_m |j, m\rangle$

have coeff. a_m numerically & analytically (for j>>1) → Verify: $\lim_{j\to\infty} \epsilon = 0$

CRSS exist: Asymptotic eigenstates of $\hat{J}$

1. We found CRSS ansatz state

$$|\psi\rangle = \sum_{m=-j}^{m_+} a_m \,|\, j\,, m\rangle$$



2. State minimizes the error

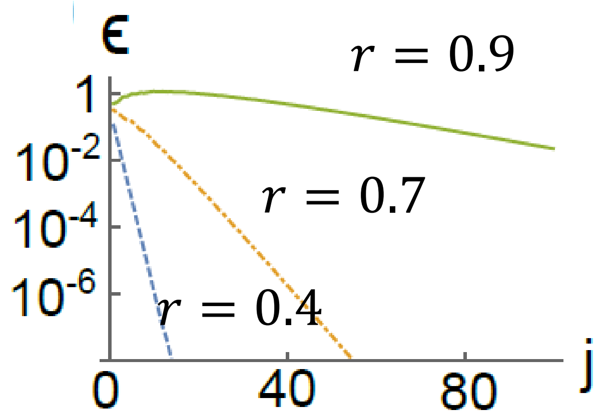
$$\epsilon(r,j) = \left\| \hat{J}|\psi\rangle - jre^{i\varphi}|\psi\rangle \right\| \sim e^{-g(r)j}$$

α ($= jre^{i\varphi}$)

For given $r = \frac{|\alpha|}{j} < 1$

error decays ~ exponentially for $j \gg 1$

→ satisfies $\lim_{j\to\infty} \epsilon = 0$ → CRSS!

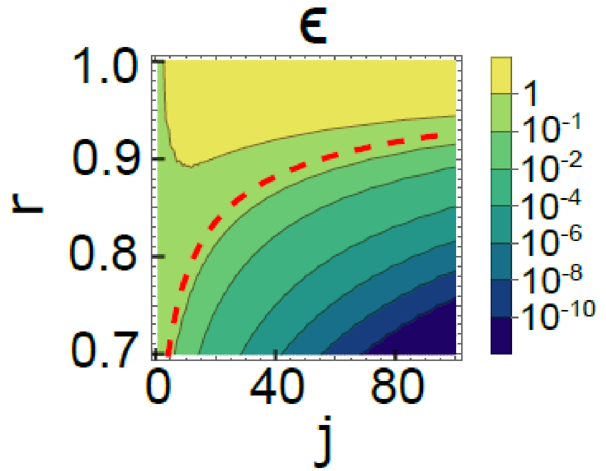


For given finite $j \gg 1$ define "Range of validity" $r < r_j$

via the conditon $\epsilon(r_j, j) = 1/e$

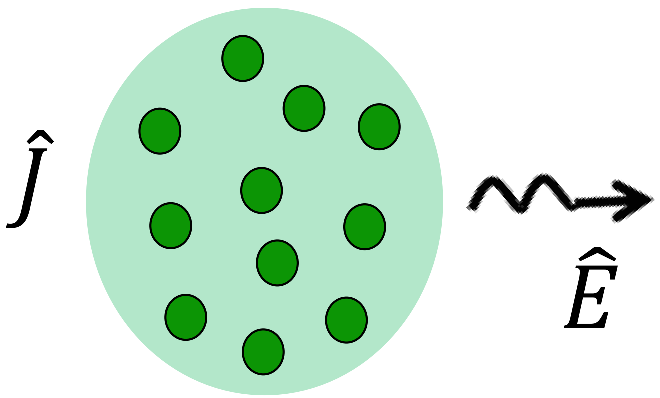
→ For $r < r_j$ our ansatz state ≈ CRSS

We find asymptotically: $r_j \sim \sqrt{1 - \left(\frac{3}{2j}\right)^{\frac{2}{3}}}$



Outline

CRSS = coherently radiating spin states $\hat{J}|\alpha\rangle = \alpha|\alpha\rangle$



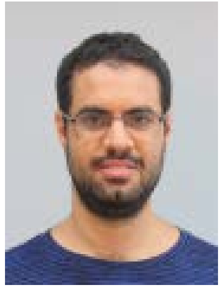
1. CRSS exist: eigenstates of $\hat{J}$

2. CRSS are physical: underlie steady-state of superradiance

3. CRSS are entangled: spin squeezing $N^{-1/3}$

4. CRSS radiate classically: dipole-projected squeezing

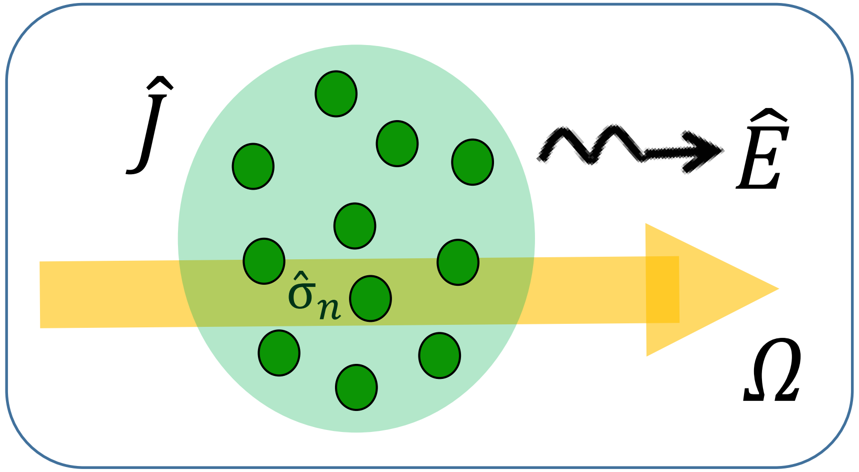
5. outlook



Ori Somech
(MSc student)

CRSS are physical: Steady-state superradiance

Resonant laser drive Ω + collective dissipation to photon reservoir $\hat{E}$



→ Master equation for atoms/macro-spin

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\left(\hat{H}_{\mathrm{nh}}\,\hat{\rho} - \hat{\rho}\hat{H}^{\dagger}_{\mathrm{nh}}\right) + \gamma \hat{J}\hat{\rho}\hat{J}^{\dagger},$$

$$\hat{H}_{\mathrm{nh}} = \hbar\left(\Delta - i\frac{\gamma}{2}\right)\hat{J}^{\dagger}\hat{J} - \hbar\left(\Omega\hat{J}^{\dagger} + \Omega^{*}\hat{J}\right)$$

γ = collective decay

Δ = dipole-dipole shift

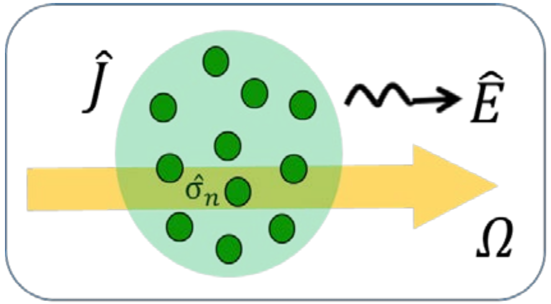
Lindblad form master eq.:

Steady state is a pure state iff it is eigenstate of $\hat{J}$ and $\hat{H}_{\mathrm{nh}}$

→ CRSS is e.s. of $\hat{J}$ $\quad \hat{J}|\alpha\rangle = \alpha|\alpha\rangle$

→ CRSS is e.s. of $\hat{H}_{\mathrm{nh}}$ for $\alpha = \dfrac{\Omega}{\Delta - i\gamma/2}$ → CRSS underlies driven-dissipative superradiance

CRSS are physical: Steady-state superradiance



$$\alpha = \frac{\Omega}{\Delta - i\gamma/2} = \langle \hat{J} \rangle$$

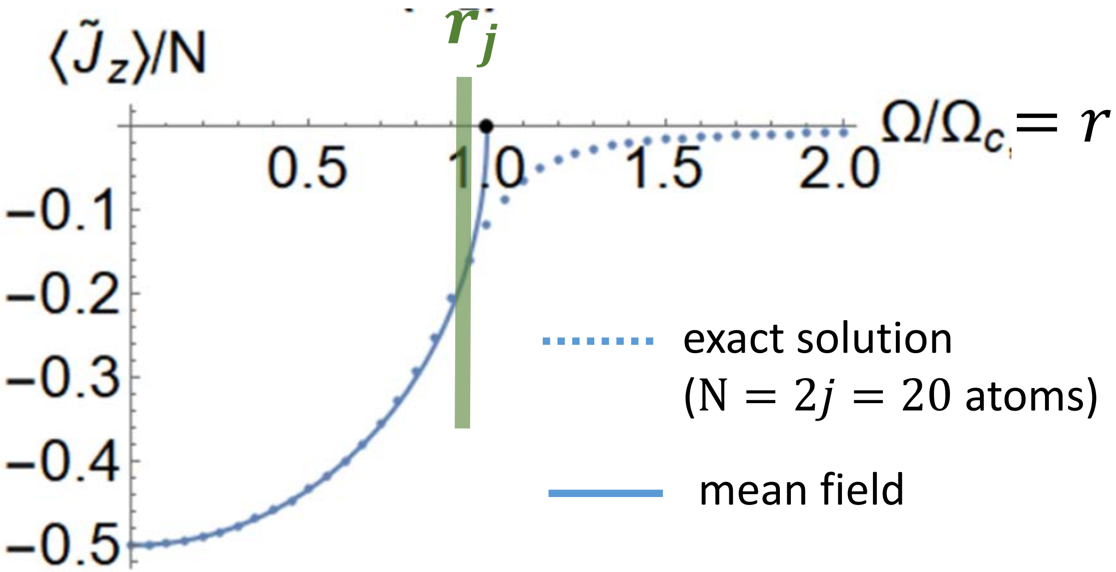
Example: dissipative Dicke phase transition

Mean-field theory prediction ("magnetization"):

$$\frac{\langle \hat{J}_z \rangle}{N} \approx -\frac{1}{2}\sqrt{1 - \frac{|\Omega|^2}{\Omega_c^2}}$$

→ Phase transition for

$$\Omega > \Omega_c = (N/4)\sqrt{\gamma^2 + 4\Delta^2}$$



CRSS theory prediction: identical!

$$\frac{\langle \hat{J}_z \rangle}{N} \approx -\frac{1}{2}\sqrt{1 - r^2} \;; \quad r = \frac{|\alpha|}{j} = \frac{|\Omega|}{\Omega_c}$$

→ Phase transition for $r > 1$ where CRSS cease to exist

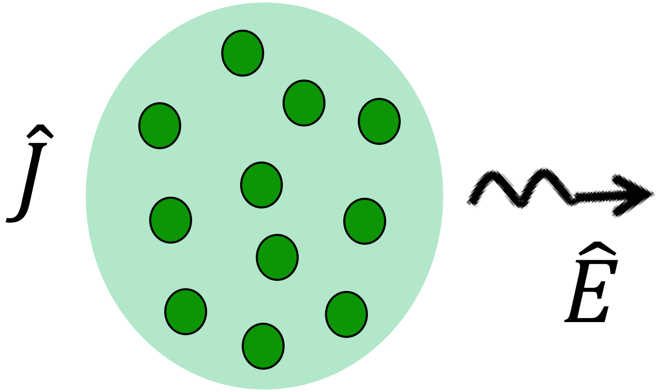
→ Existence of CRSS underlies Dicke phase transition

Finite j CRSS prediction:

crossover at "Range of validity" $r < r_j$

Outline

CRSS = coherently radiating spin states $\hat{J}|\alpha\rangle = \alpha|\alpha\rangle$



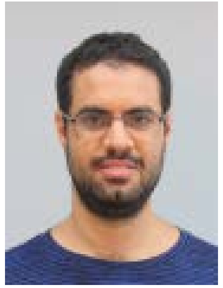
1. CRSS exist: eigenstates of $\hat{J}$

2. CRSS are physical: underlie steady-state of superradiance

3. CRSS are entangled: spin squeezing $N^{-1/3}$

4. CRSS radiate classically: dipole-projected squeezing

5. outlook

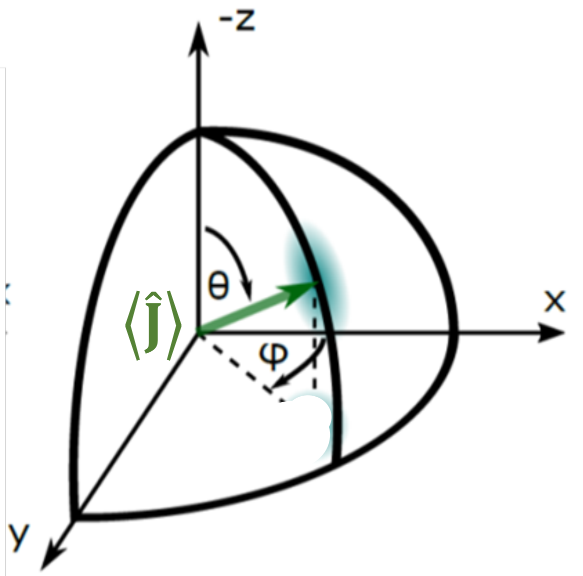


Ori Somech
(MSc student)

CRSS are entangled: Spin squeezing

Spin vector $\hat{\mathbf{J}} = (\hat{J}_x, \hat{J}_y, \hat{J}_z)$

Mean spin: $\langle\hat{\mathbf{J}}\rangle = |\langle\hat{\mathbf{J}}\rangle| \, (\sin\theta\cos\varphi, \sin\theta\sin\varphi, -\cos\theta)$



Spin squeezing parameter

unit vector ⊥ to mean spin

$$\xi^2 = \frac{N}{|\langle\hat{\mathbf{J}}\rangle|^2} \min_{\mathbf{n}_\perp} \mathrm{Var}[\hat{J}_{\mathbf{n}_\perp}] \qquad \hat{J}_{\mathbf{n}_\perp} = \mathbf{n}_\perp \cdot \hat{\mathbf{J}}$$

→ Q metrology, sensing

"standard quantum limit"

$$\xi^2 = 1$$

CSS= Coherent spin states

$$|\theta, \phi\rangle = \bigotimes_{l=1}^{N} \left[\cos\left(\frac{\theta}{2}\right)|0\rangle_l + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle_l\right]$$

independent atoms ("classical" spin)

$$|\langle\hat{\mathbf{J}}\rangle| = j = N/2$$

Spin squeezing

$$\xi^2 < 1$$

→ Q-enhanced metrology

→ Pairwise entanglement (btwn atoms)

Heisenberg limit:

$$\xi^2 \geq 1/N$$

CRSS: mean spin

$$\langle\hat{J}_x - i\hat{J}_y\rangle = \langle\hat{J}\rangle = \alpha = jre^{-i\varphi}$$

$$\langle\hat{J}_z\rangle = -j\sqrt{1-r^2} \;;$$

$$r = \sin\theta \qquad |\langle\hat{\mathbf{J}}\rangle| = j = N/2$$

$$\xi^2 = ?$$

CRSS are entangled: Spin squeezing

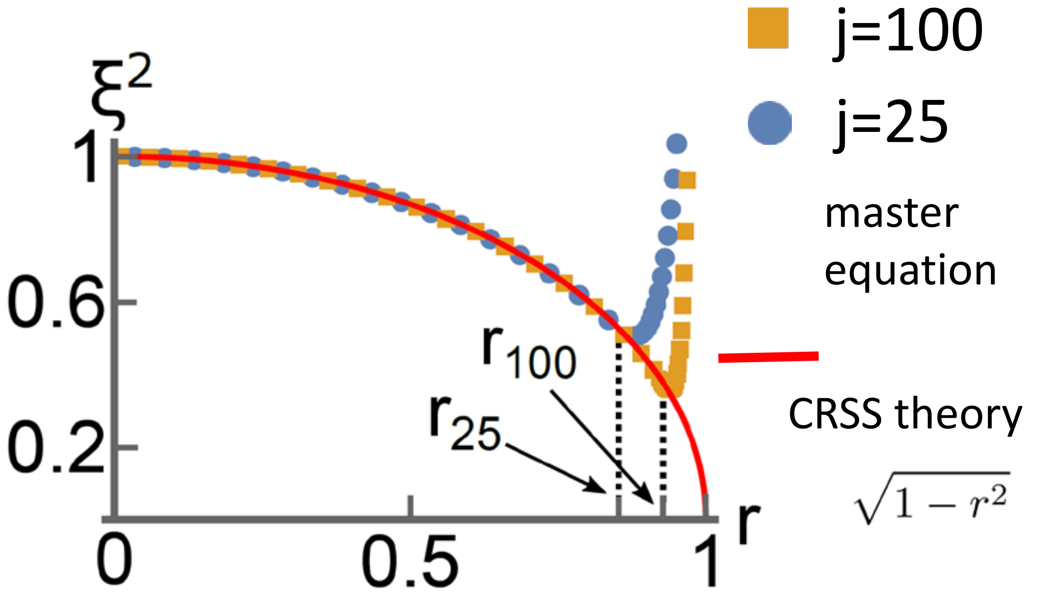
CRSS theory:

Analytical result: $\xi^2 = \sqrt{1-r^2}$

Finite $j \gg 1$ validity range: $r < r_j \qquad \epsilon(r_j, j) = 1/e$

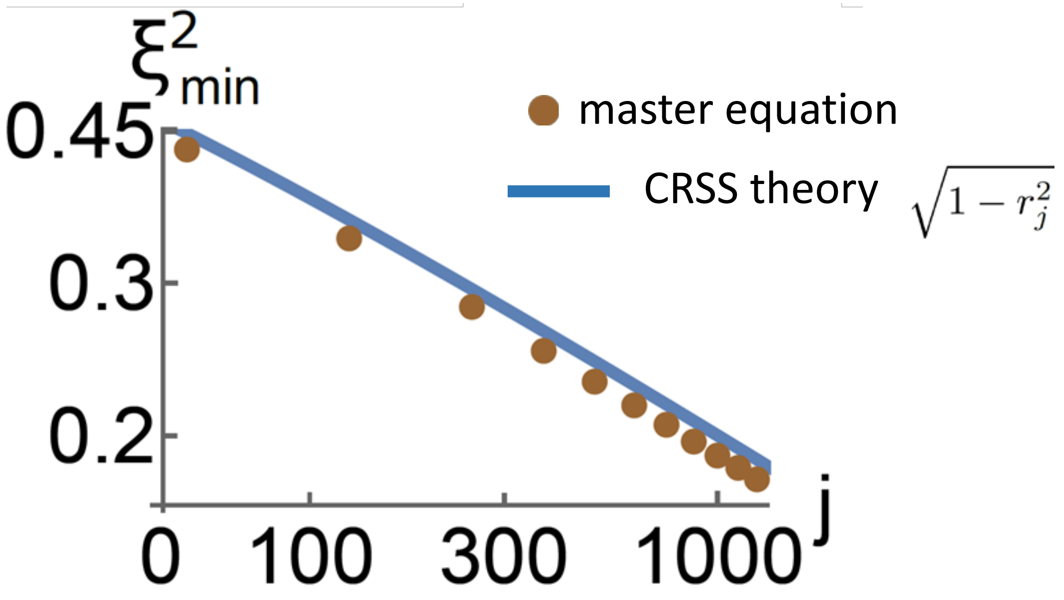
→ optimal squeezing (within CRSS theory, finite $j = N/2$):

$$\xi^2_{\min}(j) = \sqrt{1-r_j^2}$$



Steady-state superradiance:

solve Dicke master equation exactly for j=25, 100



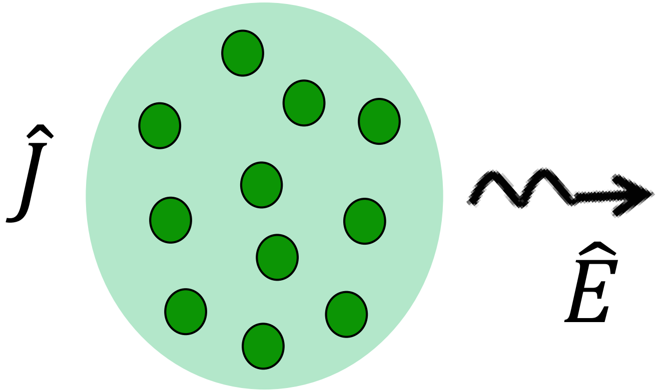
→ CRSS theory well predicts squeezing in superradiance

Asymptotic scaling (CRSS theory):

$$\xi^2_{\min}(j) \sim \left(\frac{3}{2j}\right)^{\frac{1}{3}} \propto N^{-\frac{1}{3}}$$

Outline

CRSS = coherently radiating spin states $\hat{J}|\alpha\rangle = \alpha|\alpha\rangle$



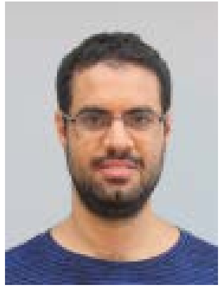
1. CRSS exist: eigenstates of $\hat{J}$

2. CRSS are physical: underlie steady-state of superradiance

3. CRSS are entangled: spin squeezing $N^{-1/3}$

4. CRSS radiate classically: dipole-projected squeezing

5. outlook



Ori Somech
(MSc student)

CRSS radiate classically

"input-output" relation to total field

$$\hat{H} = \hat{J}^\dagger \hat{E} + \hat{E}^\dagger \hat{J} \longrightarrow$$

$$\hat{E}(t) = \hat{E}_0(t) + G\hat{J}(t)$$

input vacuum field — field scattered by dipole

Steady-state superradiance:

Dipole in pure state (CRSS) → total dipole+field state separable

$$|\psi(t)\rangle = |j, \alpha\rangle_d \otimes |\chi\rangle_f$$

CRSS — field state

Prove $|\psi(t)\rangle$ is coherent state for field:

Use input-output relation + CRSS property $\hat{E}(0)|\psi(t)\rangle = G\alpha|\psi(t)\rangle$ → Coherent state of the field

...as expected: "eigenstate of dipole radiates coherent light"

$$\hat{D}|\psi(t)\rangle = \alpha|\psi(t)\rangle \longrightarrow \hat{E}|\psi\rangle \propto |\psi\rangle$$

Dicke superradiance pumps the system to a dipole eigenstate (CRSS) → coherent light radiation

CRSS radiate classically

- Why a <u>nonlinear</u> spin-squeezed dipole scatters light classically? (like a linear system) $\alpha = \frac{\Omega}{\Delta - i\gamma/2} = \langle \hat{J} \rangle$
- Why not, e.g., squeezed light?

$\widehat{H} = \hat{J}^\dagger \hat{E} + \hat{E}^\dagger \hat{J}$ → Light only "feels" $\hat{J} = \hat{J}_x - i\,\hat{J}_y$

→ Focus on noise projected to x,y plane

Field quadrature $\hat{E}_\phi = e^{i\phi}\hat{E} + e^{-i\phi}\hat{E}^\dagger$

dipole quadrature $\hat{J}_\phi = (e^{i\phi}\hat{J} + e^{-i\phi}\hat{J}^\dagger)/2$

Relation between noises: $\mathrm{Var}[\hat{E}_\phi] = 1 + 4G^2\left(\mathrm{Var}[\hat{J}_\phi] + \frac{1}{2}\langle \hat{J}_z \rangle\right)$

→ For light squeezing $\mathrm{Var}[\hat{E}_\phi] < 1$ need:

"dipole-projected squeezing" $\mathrm{Var}[\hat{J}_\phi] < |\langle \hat{J}_z \rangle|/2$

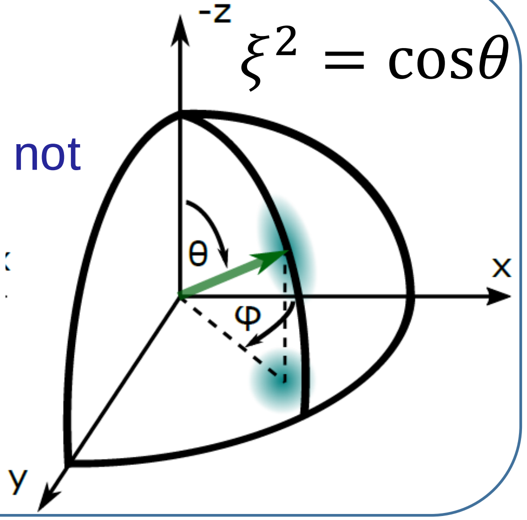
$\mathrm{Var}[\hat{J}_\phi]\mathrm{Var}[\hat{J}_{\phi+\frac{\pi}{2}}] \geq |\langle \hat{J}_z \rangle|^2/4$

<u>CRSS</u>

$\xi^2 = \cos\theta$

spin squeezed, "dipole" not

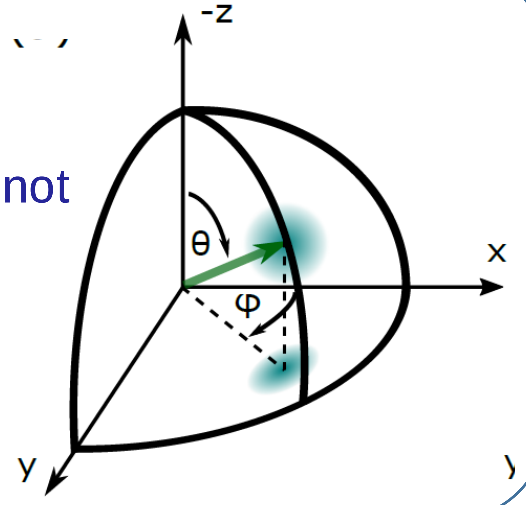
→ spin quantum
light classical



<u>CSS</u>

"dipole" squeezed, spin not

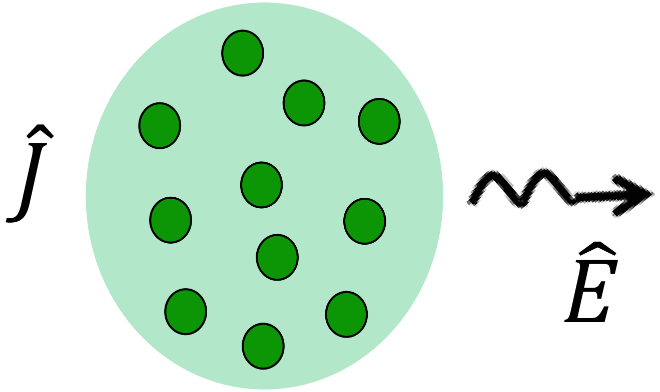
→ light quantum
spin classical



Outlook

New family of collective spin states:

CRSS = coherently radiating spin states $\hat{J}|\alpha\rangle = \alpha|\alpha\rangle$



1. Underlies Dicke superradiance

→ New tool to study superradiance phenomena: superraidant lasers, beynd permutation symmetry?

→ Predictions for cavity experimemets

2. The nature of CRSS

→ New class / scaling of spin squeezed states? $N^{-1/3}$

→ Can be produced by Hamiltonian unitaries?

3. CRSS in quantum magnetism?

→ Underlies Hamiltonian quantum phase transitions?

→ Appears in certain spin models?

Ori Somech

(MSc student)

arXiv: 2204.05455