Home Work 1

1. Chap.1 (worked example). Consider a particle in a box which extends from 0 to L. Take the initial state of the particle to be

$$\Psi(x,0) = \sqrt{\frac{2}{L}}\sin(\frac{\pi x}{L}).$$

At time 0, let the wall at L be removed, so that the particle is now in a box which extends from 0 to 2L.

   a. Show that the initial state of the particle, $\Psi(x,0)$, is normalized.

   b. Write down the eigenvalues, E_n, and eigenfunctions, $\psi_n(x)$, for the box of length 2L.

   c. Expand the initial state $\Psi(x,0)$ in terms of the complete set of the eigenfunctions of larger box, $\psi_n(x)$. Find a closed expression for the coefficients, a_n, and check that $\sum_n |a_n|^2 = 1$. Pay special attention to a_2.

   d. Write down the general expression for the moving wavepacket, $\Psi(x,t)$, in terms of a sum over the $\psi_n(x)$ with the appropriate coefficients.

   e. Find the fundamental period of $\Psi(x,t)$, i.e. the smallest τ that $\Psi(x,\tau) = \Psi(x,0)$.

   f. Draw a set of pictures of $\Psi(x,t)$ for t=0, $\tau/4$, $\tau/2$, $3\tau/4$. [Hint: use the fact that at $\tau/2$ all of the time-dependent phases are equal to their initial value except for one]. Interpret your pictures in classical terms.

   g. Optional:
   Find $<x>$ and $<p>$ for $t = 0$, $\tau/4$, $\tau/2$ and $3\tau/4$.