Decay rate and Rabi splitting in a cavity

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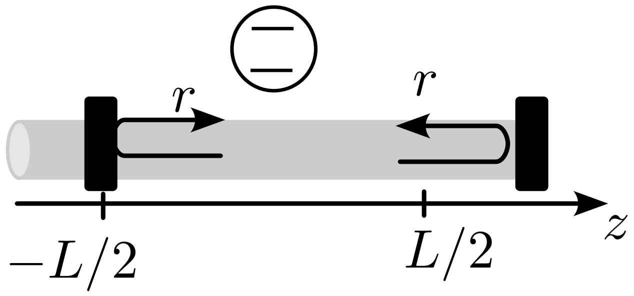

FIG. 1 Schematics of an emitter in a cavity

Consider emitter in a symmetric cavity. The cavity has two mirrors with reflecton coefficients $r_{\rm mirror}$. We have shown in the previous problems that the complex resonance frequency of the emitter is determined by the equation

$$\frac{\omega-\omega_0+{\rm i}\gamma}{\gamma_{\rm 1D}}=-{\rm i}\frac{1+r_{\rm mirror}{\rm e}^{{\rm i}\varphi}}{1-r_{\rm mirror}{\rm e}^{{\rm i}\varphi}}\,, \tag{1}$$

where $\varphi=\omega d/c$ is the phase gained by light between the mirror, $\omega_0,\gamma_{\rm 1D},\gamma$ are the emitter resonance frequency, radiative and nonradiative decay rates.

Assume that the frequency dependence of the reflection coefficient can be approximated as

$$r_{\rm mirror}{\rm e}^{{\rm i}\varphi}=\sqrt{1-\frac{1}{Q}}\exp\left({\rm i}\frac{\omega-\omega_{\rm cav}}{\Delta\Omega}\right) \tag{2}$$

where $Q\gg 1$ is the cavity quality factor, $\omega_{\rm cav}$ is the cavity resonance frequency and $\Delta\Omega\gg\gamma_{\rm 1D}$ is a parameter depending on the distance between the mirrors. If the cavity mirror reflection coefficient does not depend on frequency, we have just $\omega_{\rm cav}=\pi c/d$ for the fundamental mode, corresponding to $d=\lambda/2$ cavity thickness and $\Delta\Omega=c/d=\omega_{\rm cav}$.

Goal 1. Assume that $\Delta\Omega\gg\gamma_{\rm 1D}$ and $Q\gg 1$. Obtain the analytical answer for the solution of Eq. (1) by neglecting the dependence of the right-hand-side on ω (Markovian approximation). How much is the radiative decay rate of the emitter enhanced by the cavity?

Goal 2. Assume that $\Delta\Omega \gg \gamma_{1\mathrm{D}}$ and $Q \gg 1$. Obtain the analytical answer for the two solutions of Eq. (1) without neglecting the dependence of the right-hand-side on ω (beyond Markovian approximation). What is the physical interpretaton of these two solutions?