## Reflection from a periodic array of resonant scatterers vs N.

(Dated: February 11, 2025)

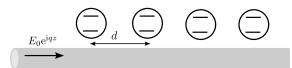


FIG. 1 Periodic array of resonant scatterers coupled to the waveguide.

We consider wave reflection from a periodic array of scatterers, shown in Fig. 1. The amplitude light reflection coefficient  $r_N(\omega)$  can be found from the following equations (Sheremet *et al.*, 2023):

$$t_N(\omega) = \frac{\tilde{t}_1 \sin K}{\sin NK - \tilde{t}_1 \sin(N-1)K} \tag{1}$$

where

$$\widetilde{r}_1 = e^{iqd}r_1, \quad \widetilde{t}_1 = e^{iqd}t_1, \quad t_1 = 1 + r_1, \quad r_1 = \frac{i\gamma_{1D}}{\omega_0 - \omega - i(\gamma_{1D} + \gamma)}$$
 (2)

are the reflection and transmission coefficients from one emitter,  $q = \omega/c$  is the light wave vector and

$$\cos K = \cos qd - \frac{\gamma_{\rm 1D}}{\omega_0 - \omega - i\gamma} \sin qd \,. \tag{3}$$

is the Bloch vector.

## Goal:

• Derive the effective medium approximation, under which for  $\gamma \gg \gamma_{1\rm D}$  and  $\omega d/c \ll 1$  one has

$$|t_N(\omega)|^2 = e^{-OD}, \quad OD = \frac{2N\gamma\gamma_{1D}}{(\omega - \omega_0)^2 + \gamma^2}.$$
 (4)

• Plot the set of transmission spectra  $|t_N(\omega)|^2$  vs normalized frequency  $(\omega - \omega_0)/\gamma_{1D} \in [-5...5]$  for  $\omega_0 d/c = \pi$ ,  $\gamma_0/\omega_0 = 2 \times 10^{-2}$  and  $\gamma/\gamma_{1D} = 0.1$  (a) and  $\gamma/\gamma_{1D} = 2$  (b). and N = 1, 2...10, 20 (spectra for same  $\gamma$  have to be on the same plot).

## Answer:

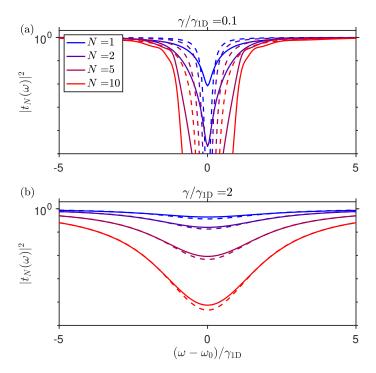


FIG. 2 Transmission spectra from the periodic structure for different values of N. Calculation has been performed for the spacing  $\omega_0 d/c = \pi$ ,  $\gamma_0/\omega_0 = 2 \times 10^{-2}$  and  $\gamma/\gamma_{1D} = 0.1$  (a),  $\gamma/\gamma_{1D} =$ 2 (b). Solid lines have been calculated exactly, dotted lines correspond to the optical density approximation, Eq. (4)). The calculation demonstrations that the OD approximation works well for a large ratio  $\gamma/\gamma_{1D}$ .

## References

Sheremet, A. S., M. I. Petrov, I. V. Iorsh, A. V. Poshakinskiy, and A. N. Poddubny, 2023, Rev. Mod. Phys. 95, 015002.