Ensemble-Averaged Quantum Correlations between Path-Entangled Photons Undergoing Anderson Localization

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We measure ensemble-averaged quantum correlations of path-entangled photons, propagating in a disordered lattice and undergoing Anderson localization. These result in intriguing patterns, which show that quantum interference leads to unexpected dependencies of the location of one particle on the location of the other. These correlations are shared between localized and nonlocalized components of the two-photon wave function, and, moreover, yield information regarding the nature of the disorder itself. Such effects cannot be reproduced with classical waves, and are undetectable without ensemble averaging.

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A quantum particle hopping between sites on a perfect lattice, such as a free electron in a crystal, performs a quantum walk [1]. The interference between the many routes the particle can take leads to ballistic spreading—the particle moves away from its initial position at a constant rate. It was assumed for many years that in a disordered lattice, where wave interference is randomized, ballistic expansion would become diffusive. However, as Anderson showed in his seminal 1958 work, such interference in disordered lattices often leads to the complete arrest of expansion, and the particle remains localized near its initial position [2]. Although first predicted in the context of quantum physics, Anderson localization is primarily a wave phenomenon, and as such it affects any form of wave. Indeed, experiments directly observing the localized wave function have been performed with light [3–6] and with matter waves [7], as well as with acoustics [8].

As a wave effect, Anderson localization can be demonstrated with classical sources of light, for example, in lattices of coupled waveguides [6]. Single particle transport essentially follows the trajectories of classical waves; hence, even when single photons are launched into such a lattice, no new phenomena are expected. In order to examine quantum behavior beyond that present in wave mechanics, one must consider the transport of more than one particle, where quantum statistics enter into play.

Only recently has attention been given to the issue of the simultaneous quantum walks of two or more particles in lattices. Theory and experiments with two identical photons propagating simultaneously in a periodic lattice showed that their fate is strongly correlated—the output position of one particle depends in a nontrivial and often surprising way on the output position of the other [9–11]. It was then natural to investigate if these correlations survive also in disordered lattices [12,13], as well as in other disordered systems [14]. Theoretical and numerical investigations were carried out, which showed that, indeed, two entangled photons remain strongly correlated even in the presence of strong disorder. In fact, in some cases, the predicted quantum correlations display intriguing, unexpected patterns.

In this work, we experimentally confirm these predictions, measuring these patterns for the first time. We note that these experiments pose a double challenge; first, to investigate correlation, it is essential to launch the quantum particles pair by pair and to record their eventual locations. As such, one can no longer send strong multiparticle beams as in previous single-particle experiments, and the correlations need to be assembled from measuring many such pair propagations. But on top of that, as in any study of randomness, it is necessary to repeat the experiment with many realizations of disorder to extract the average correlation function. Previous works have managed to achieve only one of these requirements simultaneously [15–17], while other experiments meeting both have focused on scattering from a random medium as opposed to localization within it [18–20]. In particular, Crespi *et al.*. who observed propagation of entangled photon pairs in a disordered lattice [16], did so using only a single realization of disorder and therefore did not have access to the underlying average correlation function.

The system used to obtain these results is composed of arrays of evanescently coupled, single-mode waveguides, laser written in a bulk glass slide [21–23], as schematically depicted in Fig. 1(a). In such arrays, each waveguide acts as a site on the lattice, and the evanescent coupling between waveguides allows tunneling of photons between sites. The resulting dynamics are those of a quantum tight-binding model in a static one-dimensional lattice, as expressed by the Heisenberg equation [9]:

$$i\frac{\partial a_q^\dagger(z)}{\partial z} = \beta_q a_q^\dagger(z) + C_{q-1,q}a_{q-1}^\dagger(z) + C_{q,q+1}a_{q+1}^\dagger(z),$$

where z is the coordinate along the direction of propagation and takes the place of propagation in time, $a_q^\dagger$ is the creation

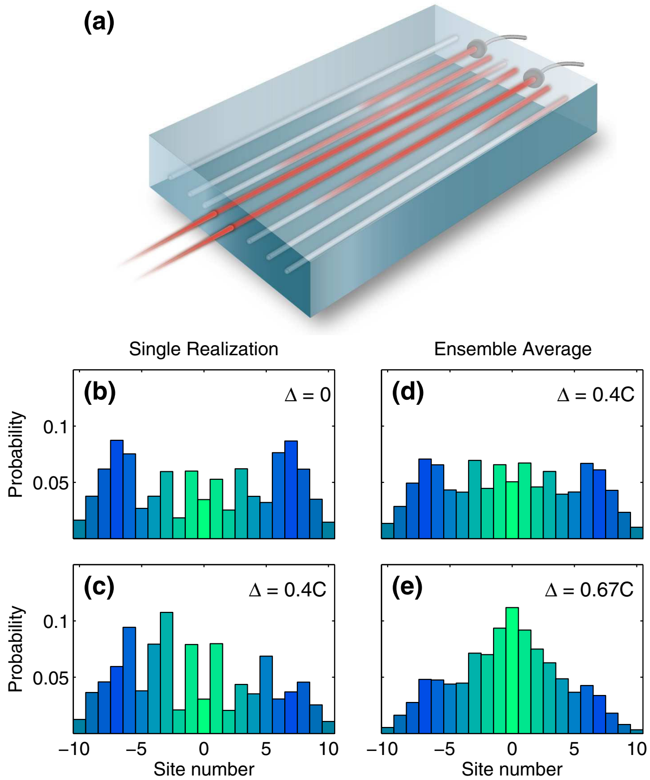

FIG. 1 (color online). Propagation of light in a disordered lattice. (a) Schematic of a disordered evanescently coupled waveguide array. Light is inserted into one or more input sites, tunneling between waveguides along propagation, and detected at the output. For a full description of the experimental setup, see Supplemental Material [24]. (b)–(e) Experimentally obtained particle density distributions after propagation, with light input at site 0. The area around the origin is highlighted in light green, while the lobe regions are highlighted in darker blue. (b) No disorder ($\Delta = 0$), single realization—a perfectly periodic lattice displaying discrete diffraction. (c) Moderate disorder ($\Delta = 0.4C$), single realization. The discrete diffraction pattern is lost, and the output does not show any discernible structure. (d) Moderate disorder ($\Delta = 0.4C$), averaged over 30 realizations. The light is now partially localized, leading to approximately 50% staying near the origin and 50% reaching the lobes. This is the regime in which the quantum correlation measurements were performed. (e) Stronger disorder ($\Delta = 0.67C$), averaged over 30 realizations. Most of the light localizes in the center, while the lobes nearly disappear.

operator for a photon in site number q, β_q is the propagation constant in site q, and $C_{q,r}$ is the tunneling amplitude between sites q and r. In terms of observables, the quantities of interest are the particle density, $n_q(z) = \langle a_q^\dagger(z) a_q(z)\rangle$, which is the probability of detecting a photon in site q, and the photon-number correlation function $\Gamma_{q,r}(z) = \langle a_q^\dagger(z) a_r^\dagger(z) a_r(z) a_q(z)\rangle$, which relates to the probability of detecting one photon in site q and the other in site r. The $\langle\cdot\rangle$ here represents ensemble averaging for both repeated two-photon events in a particular realization of disorder, and for different such realizations. Such systems have been used to demonstrate exponential Anderson localization [4,5], an essential first step on which we build in this work.

We begin by examining the effect of disorder on the particle density distribution, by comparing the output of a disordered lattice with that of an ordered one. For an ordered, periodic lattice of identical waveguides, that is $C_{q,q\pm1} = C$, and $\beta_q = \beta$, a photon propagating from a single input site undergoes a quantum random walk. This results in ballistic expansion, forming a pattern known as discrete diffraction [25,26]—where most of the density distribution ends up in two strong lobes equidistant from the input site, located at the edge of the distribution [Fig. 1(b)]. Adding disorder to the lattice interferes with this ballistic behavior. Disorder was added by randomizing the distances between the waveguides in an appropriate way, such that the tunneling amplitudes $C_{q,q\pm1}$ are drawn from a uniform random distribution in the range $C \pm \Delta$—a configuration known as off-diagonal disorder [27].

When inserting light into a single site of such a disordered lattice, the exact particle density distribution at the output cannot be predicted without knowledge of the particular realization of the disorder: different realizations will have significantly different output distributions [see Fig. 1(c)]. However, when the experiment is repeated with different realizations drawn from the same probability distribution, a pattern emerges: the averaged density distribution shows increased probability of the light staying near the input site—the expected signature of Anderson localization. This effect becomes more pronounced as the disorder is increased, leading to stronger localization [Figs. 1(d) and 1(e)], as seen in previous works in these systems where strong Anderson localization has been verified [5].

Our main goal is to observe the effect of disorder on the propagation of two entangled photons, as quantified by the photon-number correlation function $\Gamma_{q,r}$. In particular, we study the propagation of path-entangled input states of the form $\frac{1}{2}(a_0^{\dagger 2} + e^{i\phi} a_1^{\dagger 2})|0\rangle$, where two photons are input in a coherent superposition of being either both in site 0 or both in site 1, with ϕ a phase in $[0, 2\pi)$. Such entangled photon pairs have been shown to exhibit intriguing properties, as compared with product input states [12]—by controlling the phase ϕ it is possible to modify the correlation features. In periodic lattices, the two photons may bunch (tend to move to the same direction, a behavior associated with bosons) or antibunch, (tend to move to opposite sides, a behavior usually associated with fermions) depending on the setting of ϕ.

The system used to launch such entangled pairs into the array, is based on spontaneous parametric down-conversion (SPDC) pumped by two coherent beams (see Supplemental Material [24]). Light emerging from the lattice was detected via coincidence measurement by two single-photon

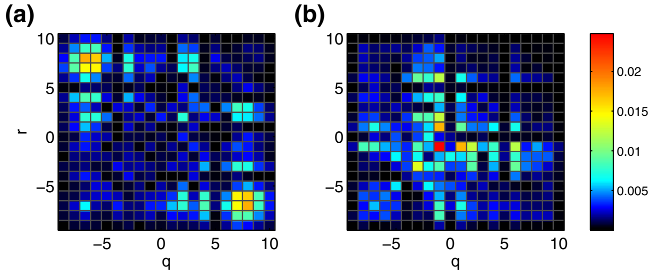

FIG. 2 (color online). Measured correlation matrices of single realizations. (a) Experimentally obtained correlation matrix ($\Gamma_{q,r}$) for the input state $\frac{1}{2}(a_0^{\dagger 2} + a_1^{\dagger 2})|0\rangle$, measured in a single realization of an ordered lattice. Antibunching between the lobes is evident. (b) The same matrix measured in a single realization of a disordered lattice. No clear pattern is apparent, demonstrating the need for ensemble averaging to obtain meaningful results.

detectors which scanned all possible output combinations. Fig. 2a shows such a coincidence map measured in a periodic lattice, with the phase $\phi = 0$. This leads to antibunching, which is marked by the two anti-diagonal peaks, signifying the high probability of the two photons to move away from each other and arrive at opposite sides of the array.

As with the density distribution, adding disorder causes these correlations to break down, producing patterns with no immediately discernible structure [Fig. 2(b)]. Averaging the correlation maps is again needed to reveal the underlying order, as the fluctuations between different measurements are significant. We have chosen the strength of disorder ($\Delta = 0.4C$), so that each photon has a similar probability to localize or to retain ballistic behavior, as can be seen in Fig. 1(d). $\Gamma_{q,r}$ was measured for two path-entangled input states: $\phi = 0$ and $\phi = \pi$, by coincidence counting between all pairs of sites at the output. The results are presented in Figs. 3(a) and 3(d), where each plot represents an ensemble averaging over 12 different realizations of disorder. This number of realizations was chosen as it supplied both experimental accessibility, and enough averaging to reliably bring forth the main features of the many-realization asymptotic behavior (as verified by statistical tests carried out on simulated data). These matrices, as opposed to those obtained from a single realization, show intriguing structure.

For $\phi = 0$ [corresponding to an input state $[|2,0\rangle_{0,1} + |0,2\rangle_{0,1}]/\sqrt{2}$, Figs. 3(a), 3(b)], if both photons remain ballistic (the corners of the matrix), they still tend to anti-bunch - exiting from different lobes. In contrast, when both localize, they tend to exit close together inside the localization area. This can be seen when we inspect the interparticle distance probability inside the localized zone, $g(\eta) = \sum_{q=-3}^{q=4} \Gamma_{q,q+\eta}$ (which corresponds to the absolute probability that one photon exited η sites away from the other), presented in Fig. 3(c). Here, we see the probability decays quickly with increasing interparticle distance.

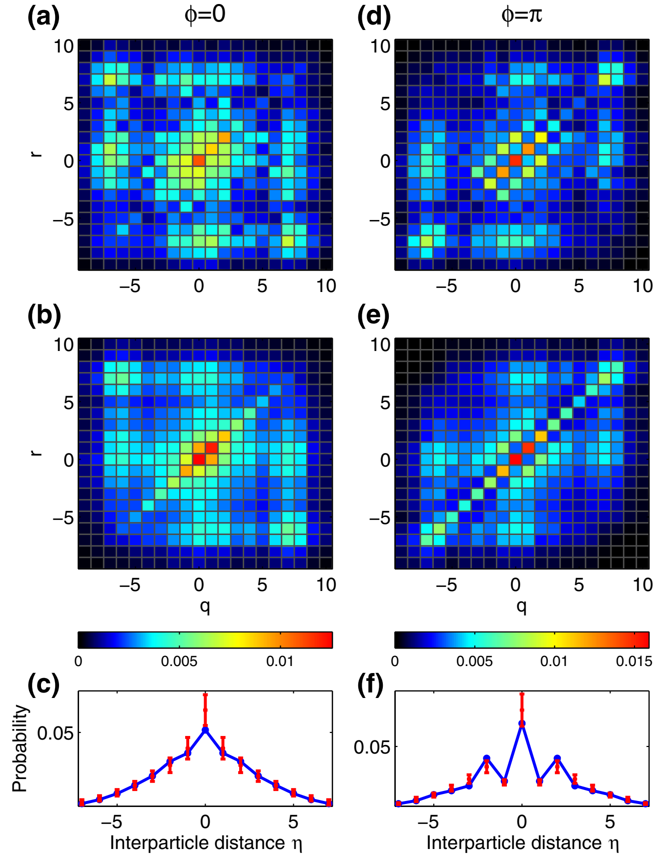

FIG. 3 (color online). Measured correlation matrices for path-entangled photons propagating in a disordered lattice, for an input state of $\frac{1}{2}(a_0^{\dagger 2} + e^{i\phi} a_1^{\dagger 2})|0\rangle$. (a) Measured and (b) simulated correlation matrix $\Gamma_{q,r}$ for $\phi = 0$. Compare with data of a single realization presented in Fig 2(b). (c) Measured (blue, solid line) and simulated (red markers) mean interparticle distance for $\phi = 0$. (d) Measured and (e) simulated correlation matrix $\Gamma_{q,r}$ for $\phi = \pi$. (f) Measured (blue, solid line) and simulated (red markers) mean interparticle distance for $\phi = 0$. Experiments are averaged over 12 realizations, while simulations were performed using 1000 realizations of disorder. Error bars of the measured mean distance plots (c),(f) are smaller than the data point markers. The simulated data in (c),(f) represent the expected results of a 12-realization set; the error bars are derived by taking 83 simulated 12-realization sets drawn from the initial 1000, and calculating the standard deviation among the results from those 83 sets.

For $\phi = \pi$ [corresponding to an input state $\frac{1}{2}(a_0^{\dagger 2} - a_1^{\dagger 2})|0\rangle$, Figs. 3(d), 3(e)], if both photons remain ballistic, they bunch, exiting from the same side of the array. If both localize, a checkered pattern emerges within the localization region. $g(\eta)$ in this case Fig. 3(f) shows peaks at $\eta = 0$ and $\eta = \pm 2$, meaning that inside the localized region, the two photons will usually exit from the same site or its next-nearest neighbor. Surprisingly, they are more likely to be separated by an empty site than to localize next to each other.

These results demonstrate that the quantum correlations of the input entangled photons manage to survive propagation through a disordered medium even while undergoing localization. The perseverance of correlations between the ballistic, nonlocalized photons might be expected, as one may consider them unaffected by the disorder. Indeed, the bunching and antibunching behavior is the same as predicted for path-entangled states in a periodic lattice [9], and they seem to persist as long as the ballistic component can still be observed. The presence of correlations among the localized photons is more surprising, considering their localization is based on the random disorder of each of the realizations. For the input state $[|2,0\rangle_{0,1}+|0,2\rangle_{0,1}]/\sqrt{2}$ (i.e., $\phi=0$), these correlations manifest as a form of bunching, or two-photon localization—the distance between photons in the localized region of the output is shorter than could be explained by classical statistics, as calculated from the cross product of the particle density distribution.

More interesting still is the checkered pattern observed for the input state $[|2,0\rangle_{0,1}-|0,2\rangle_{0,1}]/\sqrt{2}$ (that is, $\phi=\pi$). As predicted and elaborated upon in previous works [12,15], this pattern arises as a result of off-diagonal disorder, and is observed here for the first time. It stems from the nature of the localized eigenmodes of a finite lattice containing certain types of disorder: their energy spectrum contains a symmetry that originates from the spectrum of an ordered lattice. This symmetry survives when disorder is introduced into a lattice, and results in the modes having a spatial frequency of two sites. This spatial frequency cannot be observed in the particle density distribution; as each realization will have it at a different location in the array, ensemble averaging will wash it out. Nevertheless, the correlations of each realization will contain this information in a consistent form, and thus only through measuring ensemble-averaged correlations may they be experimentally verified. Such features are a unique characteristic of discrete systems, such as that originally described by Anderson, and they could not be observed in model systems that use a continuously disordered potential to study localization.

In conclusion, we have observed quantum correlations of path-entangled photons, propagating in a disordered lattice and undergoing Anderson localization. These measurements incorporate ensemble averaging over both two-photon events and different realizations of disorder—necessary prerequisites to access the two-particle localization phenomenon in its full statistical nature. They demonstrate that entanglement manages to survive propagation through a disordered medium, manifesting in a unique way distributed among the localized and ballistic parts of the two-photon state. In addition, they are capable of revealing features of the underlying mode structure, which are visible neither in the classical particle density distribution, nor in the quantum correlations of a single realization of disorder.

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