Berlin-Brandenburgische Akademie der Wissenschaften

DEBATTE

Heft 23

On the Beauty ...

Scientific meeting
Of the Assembly of Members
of the Berlin-Brandenburg Academy of Sciences and Humanities
on November 25, 2022

Berlin-Brandenburgische Akademie der Wissenschaften

Debatte

Heft 23

Herausgeberin der Reihe „Debatte“:
Berlin-Brandenburgische Akademie der Wissenschaften
Redaktion: Dr. Karin Elisabeth Becker unter Mitarbeit von Kathrin Künzel
Satz: Kathrin Künzel
Umschlagentwurf: Carolyn Steinbeck · Gestaltung
Druck: USE – Union Sozialer Einrichtungen gGmbH, Berlin

ISBN: 978-3-949455-26-1

ON THE BEAUTY …

Scientific meeting
of the Assembly of Members
of the Berlin-Brandenburg Academy of Sciences and Humanities
on November 25, 2022

Organization and moderation: Günter M. Ziegler

Contents

Preface

The abundance of scientific institutions in Germany means that once in a while (to say the least) topics are dealt with that one has already read about elsewhere and that are even dealt with so often that it is difficult to say anything new or original. The Academies of Sciences and Humanities only remain meaningful if they repeatedly explore new topics or, at any rate, discuss topics that have often been treated in a truly new way. They benefit from the fact that a wide variety of disciplines can enter into conversation with each other, because the members of the academies come from very different disciplines. Such a conversation requires time, a certain amount of experience and a certain level of knowledge of each other's subjects. The Berlin-Brandenburg Academy of Sciences and Humanities began holding interdisciplinary debates of this kind twenty years ago in the twice-yearly plenary meetings of its members, and also published (thoroughly reviewed) verbatim transcripts of these discussions. At the end of each debate, the topic of the following one is set and usually two, sometimes more, persons are appointed to recruit speakers for short inputs and to lead them through the discussion.

In November 2022, everything was a little different, as the topic of the debate had already been hatched during a visit by the President of the Berlin-Brandenburg Academy of Sciences and Humanities to the President of the Israel Academy of Sciences and Humanities in March 2022. At a luncheon in Jerusalem, the computer scientist David Harel and the historian and theologian Christoph Markschies, respectively, decided to make the age-old topic of "beauty" the subject of an interdisciplinary debate at the Berlin Academy, but very deliberately to take a lecture by their Israeli colleague as the starting point, and to ask whether, in talking about the beauty of mathematical argumentation, "beauty" is understood to mean the same thing as, for example, in describing a work of art as beautiful. The fact that the two presidents were able to agree on such an idea so quickly (before they had even finished the appetizer) was due to the fact that both had already thought about and had also published on beauty for an extended period of time.[1] In order to explore what very different disciplines in the humanities, culture, natural and social sciences mean when they use term "beauty", some members of the Berlin-Brandenburg Academy were to speak about how "beauty" is understood in their respective disciplines alongside the guest of honor David Harel. And it was also clear from the outset that Günter M. Ziegler – a member of the Berlin-Brandenburg Academy of Sciences and Humanities and President of the Freie Universität Berlin – would have to moderate this debate, as he has after all published on precisely this topic as well.[2]

The fact that the presidents of the academies in Berlin and Jerusalem jointly came up with such ideas for the Berlin plenary session is a sign of the very close relations that have existed between the Israel Academy of Sciences and Humanities and the Berlin-Brandenburg Academy of Sciences and Humanities for decades. Academics living and working in Israel are members in Berlin, and vice versa – members of the Berlin-Brandenburg Academy are also members in Jerusalem. Members of both academies meet in joint working groups and publish the results.[3] Whenever there are anniversaries, reciprocal visits also take place, a fairly regular occurrence.[4] In this respect, the debate in the run-up to Einstein Day 2022 was another jewel in a wonderful chain of encounters, lectures and symposia, which will of course be continued in 2023.

It only remains for me to first thank David Harel, and Günter M. Ziegler of course, as well as all the other speakers from the bottom of my heart for this highly interesting and stimulating debate, which of course – and this will hardly come as a surprise – led to very different answers. I am sure that there

---

1 David Harel and Yishai Feldman, Algorithmics. The Spirit of Computing, Amsterdam [3]2004; Christoph Markschies, "Die Schönheit und Europa – Europa und die Schönheit" [celebratory lecture for the 60th birthday of André Schmitz-Schwarzkopf on 26.9.2017 in the Leibniz Hall of the Berlin-Brandenburg Academy of Sciences], Berlin 2017, 7-29 (printed by the foundation "Schwarzkopf-Stiftung Junges Europa").

2 In collaboration with Martin Aigner, he has published a book-length collection of mathematical proofs of outstanding beauty: Martin Aigner and Günter M. Ziegler, "Proofs from THE BOOK" (with illustrations by Karl H. Hofmann), Heidelberg/Berlin 62018.

3 Yohanan Friedmann, Christoph Markschies (eds.), Rationalization in Religions. Judaism, Christianity and Islam, Berlin/Boston 2018; (eds.), Religious Responses to Modernity, Berlin/Boston 2021 or the conferences "Science and War – Science and Peace" in the years 2009 and 2011.

4 Christoph Markschies, "Compassion. Some Remarks on Concepts of Divine and Human Compassions in Antiquity", The Israel Academy of Sciences and Humanities. Proceedings, Volume VIII/5, Jerusalem 2011.

is no comparable publication anywhere and in this respect, the booklet presented here certainly fulfills the demand of discussing an often-negotiated topic in a completely new and quite original way.

Jerusalem, April 9, 2023 Christoph Markschies

GÜNTER M. ZIEGLER

Welcome

Meine Damen und Herren, ladies and gentlemen, welcome to this session, bienvenue, welcome, willkommen. It is great to have you here for the scientific part of the meeting of the Academy, where we have chosen the topic of beauty to be at the center of everything we discuss.

And in that context, it may be appropriate for me to start with a poem by Hans Magnus Enzensberger, especially as, on my way to this meeting, I got the message – or rather the message got me – that Enzensberger died this week. He was an outstanding writer, poet, and intellectual. To honor him, here is one of his poems, in both the German original and in an English translation:

Two Errors

I must admit that on occasion
I have shot sparrows at cannons.

There was no bull's eye in that,
which I understand.

On the other hand, I never claimed
that one must remain completely silent.

Sleeping, inhaling, making poetry:
this is nearly not a crime.

Remaining completely silent
of the well-known discussion about trees.

Cannons against sparrows,
that would be to lapse into the inverse error. [5]

Zwei Fehler

Ich gebe zu, seinerzeit
habe ich mit Spatzen auf Kanonen geschossen.

Daß das keine Volltreffer gab,
sehe ich ein.

Dagegen habe ich nie behauptet,
nun gelte es ganz zu schweigen.

Schlafen, Luftholen, Dichten:
das ist fast kein Verbrechen.

Ganz zu schweigen
von dem berühmten Gespräch über Bäume.

Kanonen auf Spatzen, das hieße doch
in den umgekehrten Fehler verfallen.

---

[5] Hans Magnus Enzensberger (1929-2022): Zwei Fehler, from: Gedichte 1955-1970, Suhrkamp, Frankfurt/M. 1971; English translation by Tom von Förster, 2011, unpublished.

So much for (and by) Hans Magnus Enzensberger I remember one evening here in this very room twelve years ago, when we dedicated the "Enzensberger Star" to him. We dedicated a work of mathematical beauty to the poet. Which takes us to our transition from poetry to beauty. We go now from poetry to beauty, and I'll just ask the President of the Academy to introduce our first and main speaker for this session. Christoph?

CHRISTOPH MARKSCHIES Also, a very warm welcome from the President's desk. It is probably the first plenary meeting in our 320-year history to be held in the English language. In the 18th century, most of these plenary meetings took place in French. So, it seems to me like we are moving back to a certain standard of internationalization.
A very warm welcome. When David, the President of the Israel Academy of Sciences and Humanities, and I walked through the Israel Academy's floors in March of this year, there was a panel taking place with the foreign relations of the Israel Academy of Sciences and Humanities as part of an exhibition, and there were only two academies from Germany, both of them close friends of the Academy in Jerusalem, namely the Leopoldina and the Berlin-Brandenburg Academy of Sciences and Humanities.
After some talks, we had a wonderful lunch. And instead of talking about science policy and international relations and all these matters that presidents usually talk about, we discussed some of the questions and topics from our disciplines, and we realized that beauty and the question of what exactly beauty is in our disciplines is an interest we share.
And so, the idea was born to invite David to Berlin and to ask him to deliver a lecture to us, something that is also new in the last 30 years and definitely not the same thing we had before in the 18th century. It is very unlikely that anyone will talk in the French language, and the interdisciplinary approach was probably not really a frequently applied thing in centuries past.
I am extremely happy that David accepted our invitation, and that Günter Ziegler will be chairing the following discussion. David is not only the President of the Israel Academy of Sciences and Humanities, but he is also an expert in mathematics, software engineering, system biology, and logic. He graduated from MIT and, since 1980, he has been a professor at the Weizmann Institute of Science. There are so many other things I could now mention, David, bit I will skip them to allow more time for your lecture.

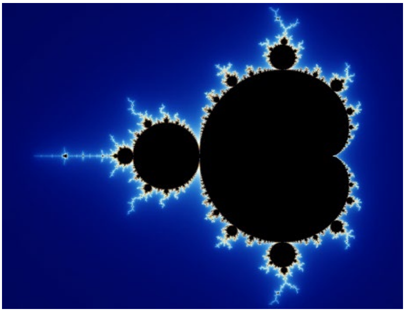
And so, you are cordially invited to deliver to us your lecture on beauty in mathematics, and I would ask the colleagues here to welcome you with a very warm round of applause. We are very happy that you can be here with us today.

Lecture

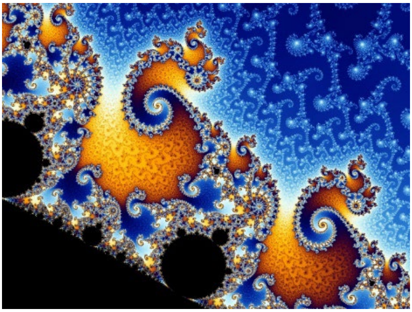
DAVID HAREL

On the Beauty of Mathematics

Thank you very much, dear Christoph, for inviting me. I am always very happy to be in Berlin, and this is a particularly happy occasion.
First, I am planning to disappoint you all. You are probably expecting this lecture to be full of things (fractals) like these:

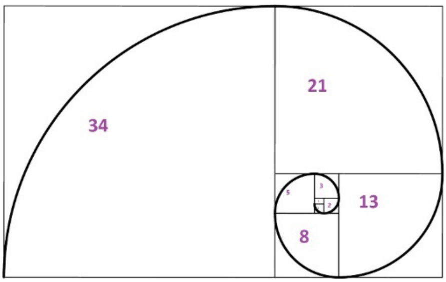


Wikipedia, Wolfgang Beyer



Wikipedia

Or very beautiful images to do with the golden ratio, such as spirals and such things, which, I am sure, many of you are familiar with:



Wikimedia Commons

Chris 73/Wikimedia Commons

But, sorry, this is not what the lecture is about. It is not necessarily about images that are very beautiful or mathematical ratios that have a lot of aesthetic applications.

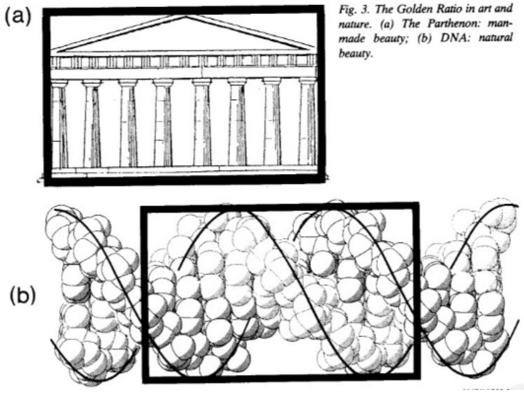


Fig. 3. The Golden Ratio in art and nature. (a) The Parthenon: man-made beauty; (b) DNA: natural beauty.

However, before I tell you what this lecture is *really* going to be about, let me make one small exception. I want to mention a paper from 1984 ("*Beauty is in the Genes of the Beholder*", Harel, Unger, Sussman, TIBS 1984), in which we showed that the DNA molecule is proportioned exactly according to the golden ratio. This is actually a serious article. It was written a little bit "tongue in cheek", half a joke, but the proportions of the golden ratio, which you can see in many pieces of art and architecture, and of course, in nature, are also almost exactly the proportions of the DNA, including the shift between one of the helices and the other.

So, this is the only part of the lecture, in which I'll be showing something that *looks* beautiful, and the rest of the lecture is going to be about completely different things.

I would like to begin with two quotations from a wonderful little book from 1940, with the title "*A Mathematician's Apology*", by Godfrey Harold Hardy, who was a very famous British number theorist. I recommend that you read it. It contains many semi-outrageous statements, including the proposition that mathematics is only there for its beauty and does not have to have applications. We can argue about that.

Hardy also said something extremely interesting: "A mathematician, like a painter or poet, is a maker of patterns." What he essentially means is that a painter makes patterns with lines, shapes, and colors, and a poet makes patterns with words and phrases. At this point, I always ask people, "With what does a mathematician make patterns?" People say: "Maybe with numbers or geometrical figures". – No, Hardy claims – and I totally agree – that a mathematician makes patterns with *ideas*. He actually adds that, if the mathematician's patterns are more permanent than those of the painter or the poet, then it is *because* they are made with ideas, adding that "the mathematician's patterns, like the painter's or the poet's, must be beautiful; the ideas like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics."

It is going to be a little difficult to convince you that this is the heart of the notion of beauty in mathematics, but I will try to do so. Of course, applied mathematicians and computer scientists like me can argue with Hardy's claim that there is no permanent place in the world for ugly mathematics, but it is nevertheless very interesting to see his purist point of view.

I am going to try to present some modest appetizers for the idea of beauty in mathematics, in order to illustrate this fundamental notion of beauty. And in most of the cases I will show that the beauty will not necessarily be visual, or numerical, or based on simple formulas. I will try to give you a feeling for that.

As we shall see later, some of the things are a little like the work of a magician. A magician does something very impressive. Sometimes, the effect is beautiful and unbelievable. But very often, it is the technique used and the mode of thinking that goes into making the piece of magic that creates the beauty. I will try to illustrate both sides of this effect-versus-method phenomenon, or in the case of mathematics, the theorem-proof notion. My basic motivation, however, will be the beauty of the ideas and how they flow into each other.

I will touch on the following areas, all of them fundamental topics of mathematics, one by one:

- Topology
- Statistics/Combinatorics
- Number Theory
- Logic
- Algorithmics
- Geometry

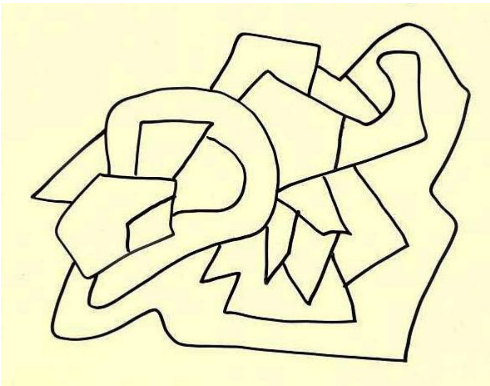
In each case, I will show you something: a little thing, a slightly larger thing, a less important but beautiful thing, or a more important thing that is beautiful as well.

Topology

Let us start with topology. I don't have time to ask the non-mathematicians here whether they have ever heard of this term. In principle, topology is like geometry, but everything is made of rubber.

In geometry, a circle and a square are two different things. In topology, they are the same because, if the circle is made of rubber, you can kind of pull it, and turn it into a square, and vice versa. In three dimensions, a sphere and a cube are the same thing topologically although they are different geometrically, because you can turn one into the other if they are made of flexible rubber. However, a donut is different even topologically, because you have to punch a hole in the sphere to get a donut, and to close up the hole in a donut to get a sphere or a cube.

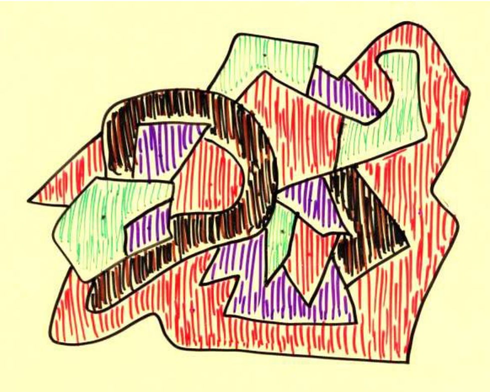
So, topology is a more *fundamental* area of mathematics, because differences in shape, size, and color, and whether you have a corner or not are immaterial. The basic issues are which things are connected, which things have holes in them, and how many, etc.



The first topology example I want to show you is called the four-color problem. You have a map of countries on the plane, and you are supposed to color this map using different colors. The only rule is that two countries that have a common border, even if it's a tiny piece of border, have to be colored using different colors. Okay?

The question of how many colors suffice to color – not just this particular example but *any* map – remained unanswered for 180 years. It is worth pausing for a second to think about that. When people see this question for the first time, they say, "What's the problem? You probably can build maps that need lots and lots of colors." Maybe you have one country over here, and then you have lots of little countries around it. They all touch that country and each other, and then you have another country coming around and touching several others … And thus, you may be able to force the number of colors to be very large.

This is not the case. For many years, it was well known that you couldn't color all maps with three colors. There are maps that require four. This is not difficult to show. And it was also known 200 years ago that five colors suffice. The



question that remained unanswered for so many years was whether *four* colors suffice. Don't ask me why this is important. These are not the kinds of issues I will touch on. And the so-called 'Four Color Theorem', which was proved in 1976 (by the way, with the aid of a computer to carry out some of the more complicated calculations), is that four colors indeed suffice. Of course, the picture on the left is not proof of that statement. It just shows that the map displayed above can indeed be colored using four colors.

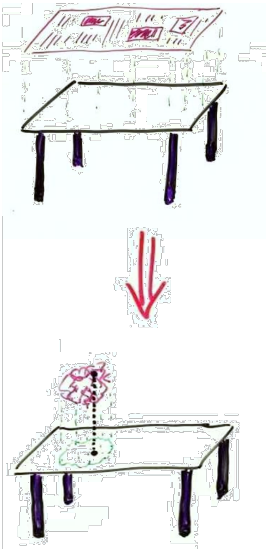
I want to make sure that you all understand why this Four Color Problem and its solution are issues in topology but not in geometry, because it does not matter what the countries look like, whether they are large or small, round or square, or look like snakes. The only thing that matters in this issue is which countries have a common border, i.e., which of them "touch" each other. And the touching, the connectability, is a topological issue.

As I said before, this is a fundamental notion in mathematics. How can you arrange things touching each other requiring more than a certain number of colors? The theorem that four colors suffice finally solved one of the most famous long-standing unresolved problems in mathematics.

I want to show you another well-known result in topology called 'Brouwer's Fixed-Point Theorem' from 1911. Here is what it would look like in a typical textbook:

Every continuous map f: Bd → Bd,
of the d-dimensional closed unit ball
to itself has a fixed point; that is,
a point x0 ∈ Bd, with f(x0) = x0.

The way we mathematicians write mathematics may look ugly, but behind what we state is something very interesting, profound, and yes – beautiful. The statement of the fixed-point theorem appears to be very boring. It doesn't look beautiful. In fact, most people don't understand it the way it is written. Even mathematicians need a little while to realize what it says. But I want to show you what this wonderful theorem actually says, and to do so I will just give you a modest special case in two dimensions.



The theorem says the following: Suppose we have a little table like this one on the left, and we have a newspaper that is exactly the shape and size of the table. We carefully put the newspaper over the table so that it does not protrude from any of the sides. It exactly covers it.

By doing that, we have associated each *point* in the newspaper – a point is a zero-sized needle-point – with its companion point on the table. This is an association between points. In the theorem statement, this is called the *map*. Okay?

Now we take this newspaper, and we do any violence we wish to do to it, but only if is allowed topologically. You are not allowed to tear it in pieces. You are not allowed to make holes in it, but you can crumple it, fold it and stretch it if it stretches, and so on. You then take the mess that results, and you put it again on top of the table, anywhere you want, except that it's not allowed to protrude outside the borderline of the table.

Now the theorem says – and again this is but a special case – that no matter how you manipulate the newspaper in a topologically allowed way, there will always be at least one point in the newspaper that will be located *exactly* above its companion point on the table.

At this point, one must pause for a few seconds because sometimes people say, "Well, obviously, because it's on top of the table." But no, it is not obvious at all because, suppose you took the newspaper and you just moved it half an inch over the side, all the points will have moved places, right? But then there's a piece of the newspaper that is outside the table. So, you take that piece, and you fold it in, and then you turn the whole newspaper around. You turn it upside down. You make another fold, and you leave it there. Why should it not be the case that by doing this I will have been able to move *all* the points from where they were?

The theorem says that no, you cannot do anything like that in a way that moves all the points to new locations. There will always be a fixed point. At least one.

Now, this is a theorem in topology, and if you are allowed to do topological violence, like tear the newspaper in pieces, the theorem is no longer true!

Imagine us taking the newspaper and tearing it right down the middle, and we then take the two halves and just switch them around. You have moved all the points on the newspaper to different places. That is possible only because you have done something to the newspaper that changes its topology. That's why this is a theorem in topology and not in geometry.

Statistics/Combinatorics

Now let me get to statistics/combinatorics. Are you interested in seeing a card trick? I am not a real magician. But I can illustrate some combinatorics using cards. Is that okay? Good.

Does anyone here know how to do a shuffle, a riffle shuffle? No? So let me just show you this normal pack of cards. You can check it out when I am done. I'll do this on my own.

I can cut the cards. I can cut the pack again. I am not a magician, and you can see that my movements are not professional. They are kind of slow. And now I am going to ask someone – maybe you? I am going to deal some cards, and you just tell me when to stop. Okay? And now since we don't have a volunteer to do the riffle shuffle, I will do the riffle shuffle myself. Of course, it would have been more impressive if someone else would have done it. But it's too late now.

What I will do next is the following. I am going to give out four cards to various people. Don't be offended if you don't get such a foursome – the only thing I ask of the people who get the four cards is this: Don't look at them; just keep them in your hand. What those who have the cards can do and actually should do is mix them up very thoroughly, very, very thoroughly. And now look at them! How many people here have exactly one heart card? How many people have exactly one diamond? How many people have exactly one spade? How many people have exactly one club? [At this point all 13 people who got four cards said yes to these four questions.]

You're allowed to clap. But you're not clapping for me. This is mathematics. Good. So, this is combinatorics. It worked, thank you.
Now comes the part of the lecture for which I need your full participation. I am going to ask a question about the audience here. The only thing I would like is for people to concentrate. And the other thing I want to ask is that you make believe that you don't know anyone else here. Or more precisely, you don't know anything *about* anyone else here; because maybe the question would be, is there someone here whose father's name starts with an S? And maybe you know that this person's father's name was Stephan, and I don't want you to know that. So, all you know are things about yourselves. Is that okay?
And I am going to actually place a money bet for the benefit of the Berlin-Brandenburg Academy. The question is: *What is the chance that there are two people in this room with their birthday on the very same day/month?* We're not getting into years here, of course. Now, I am not sure how many people are in this room, roughly 50 or 60? More than 60! Thus, I will bet 1,000 Euros, a true bet, that there are two people here with the same birthday. Maybe I should bet 750 Euros, just to be on the safe side.
By the way, I want to make sure that you realize that I am taking a real chance here. How many people have to be in this room so that I will be taking no chance whatsoever? 366, yes, because there are 365 days in the year, and we can all try to choose a different day of the year for our birthday, but then when the person numbered 366 has to choose a birthday, he or she will discover that all the days have already been taken by others. If there are fewer than 366 persons, I am taking a real chance.
Despite this fact, I have done this several times in lectures and I have never lost money. When the numbers were small, I did not bet any money, and in one case, I lost. But in the present case, I am actually betting 750 Euros. And remember, you don't know anything about anyone else.
How are we going to check this? I am going to point to people one by one, and I am going to ask you to state your birthday in terms of day and month, and everyone else has to listen very carefully. If you hear your birthday, you shout, "Bingo." You really all have to be quiet, and you have to concentrate. This is the only way I can do this in less than an hour. Okay? Everyone listening?

DAVID HAREL says April 12

FEMALE VOICE 21st March

DAVID HAREL repeats March 21st

MALE VOICE May 19

DAVID HAREL repeats May 19

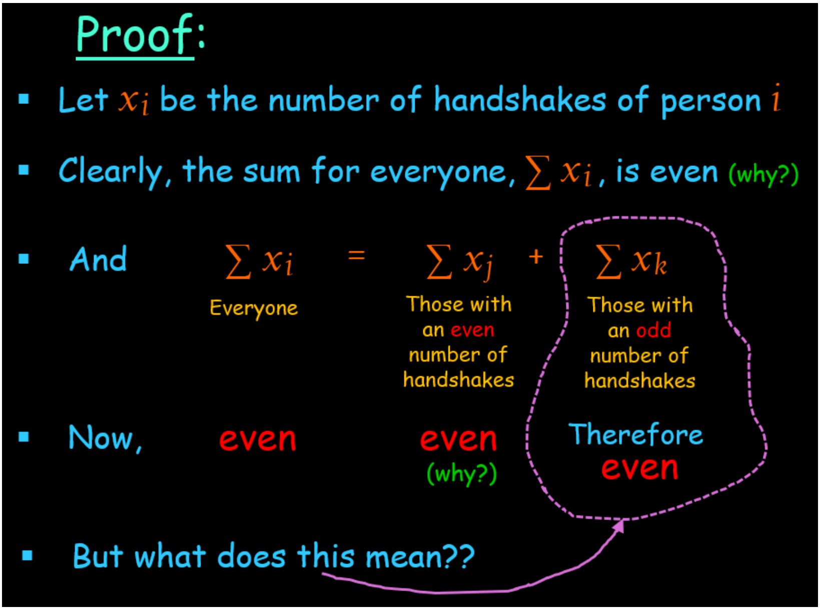
MALE VOICE Bingo!

DAVID HAREL That's it! I apologize for this happening early on because I know people really like this to take more time going around, and then very often, people say, "Let's keep going. Maybe there are two such equal-date pairs of people." But the chances that there are *two* pairs with the same birthday is much smaller, and I don't want to waste time doing this. And so, I am sorry, Christoph, no 750 Euros for the Academy. I'll give you 10 Euros later just to make you feel good.
Here are the facts. The number to remember is this: if there are 23 people (and the assumption is that they are taken from what we call a uniform distribution, which it is not exactly the case – because, people tell me that there is a slight raise in childbirth in September and October, maybe because it's cold in the winter, I don't know. But in general, if we assume a uniform distribution, and if there are 23 random people in a room, there will be a 50 percent chance that two persons have a common birthday.
By the way, sometimes, people – when I ask them what the chances are – say, "Well, maybe if there are 36 people in the room, it's a 10 percent chance." That's an egocentric answer because someone saying that is really thinking, "Hmm, is there someone here with *my* birthday?" That's not the question. We have to be happy if there are *any* two people with the same birthday.
The chances, to give further examples, are as follows: For 30 people the chance is over 70 percent; for 40 people it is over 89 percent; for 50 people it is already 97 percent. And here we are somewhere between 50 and 100; for 100 people it is still not 100 percent sure, but it's 99.99996 percent. And as the gentleman over there noted, no situation with fewer than 366 persons is 100 percent sure.
Now I want you to show what a pretty, but not very deep or not very difficult proof of a mathematical theorem looks like, because most of the time I will be talking about the results not the proof.
And what I will mention is a cute fact. So, let's say that, instead of a very serious meeting of the Berlin-Brandenburg Academy, we were at a party. And

some of us shake other people's hands. Maybe I shake 10 hands. Maybe you shake 15 hands. Maybe you shake zero hands. We all shake a certain number of hands, zero or more.
And this theorem says:

The number of people who shake an odd number of hands is even.

Sometimes people will say, "Well, of course, because each handshake contributes handshakes to two people," but that's not the point. Let me show you how this is proven.
I apologize for the Greek letters, but I am doing this deliberately to show you how we mathematicians write such a proof to each other. Here is the proof: Let's denote by x_i the number of hands person i has shaken. So, my x is 10. Your x is 15. I think yours is zero, and so on.



Let us denote the sum of all the numbers of handshakes (summed over everyone) by A. A must be even because every handshake contributes 1 to one person and 1 to another person. So, taking all these numbers together, the sum A has to be an even number.

And now we just do the following. We split the sum A that talks about everyone into two sums. One represents those like me who have shaken an even number of hands and the other covers those like you who have shaken an odd number of hands.
All the x_j in the middle summation term in the formula are even numbers, so their sum, say S, must be even. On the left side of the equation, we have an even number A, which turns out to be equal to an even number S plus some other number, say T. The only way that the sum of the two numbers S and T, one of them even, can be even is if T is also even. But what is this sum T (the one on the right-hand side of the formula)? What does it mean that the sum of the x_k over all people who have shaken 15 hands, one hand, 31 hands, and so on is even? How can a sum of odd numbers be even? This is only possible if the *number* of numbers we are summing is even!
And so we have just proved that the number of people shaking an odd number of hands is even. This is a simple, but quite beautiful argument. You don't have to count. You don't have to get in the details. You don't have to do experiments. It's common sense with a little bit of understanding about odd and even numbers. That's all.

Number Theory

Here is a very curious fact in number theory. Suppose we have an elastic band, which is 1 meter in circumference, and we have a little ant at the very top. The ant gradually and continuously moves 1 centimeter every minute. After every minute that goes by, we stretch the circle so that the circumference expands by 1 meter. So, after this poor little ant has done 1 centimeter, the entire circle becomes not 1 meter around but 2 meters around. Then the ant does another centimeter, and the circle's circumference goes from 2 meters to 3 meters, continuously expanding.
Question:

Will the ant ever reach the starting point?

People's most common answer, including that of many mathematicians is, "No way, no way." The ant is moving so slowly, and you are expanding the circle not by another centimeter, but by a whole *meter* every single minute, and all the poor ant can do in a minute is to progress a mere centimeter. But the answer is actually *yes*, believe it or not. I will not get into the details. For the mathematicians among you, it is the difference between an exponentially growing sequence of numbers and one of the sequences you know. The answer is yes. Eventually, after a long time, the ant will indeed reach the starting point again. This is a case where the effect, like the card trick, is magic. It is almost unbelievable. There are several techniques for how to prove and illustrate this, but I will not show them here.

In number theory, one of the most curious things happens when you move from finite numbers, numbers such as 3, 6, 115, to infinity. I have found over the years that most people have a problem even just thinking about infinity. They say, "There's an infinite number of idiots in the world." No, there is a large number of idiots in the world, maybe *very* large, but it is not infinite. Nothing in the real world is infinite.

But when you really think about infinity, you have to give yourself a slight additional way of diving into the notion. To illustrate this for you, I want to carry out a very simple thought experiment.

Suppose we have a very, very, *very* large basket, and it has lots and lots and *lots* of oranges in it. In fact, it has – excuse me – *infinitely many* oranges in it. Infinitely many, not a million, not a billion; infinitely many. And they are marked. There is orange number 1, orange number 2, orange number 3, etc. No matter what number you say, that orange is somewhere in the basket. So you need an infinitely large basket. Use your imagination.

What we are going to do now consists of a sequence of "steps". A step has two parts. In the first part of a step, we put our hand in the basket and take two oranges out. In the second part, we put back in the basket one of the two oranges we just took out. These two parts constitute one "step". We carry out one step, two steps, again, take out two oranges and put one back in, three steps, and so on. How many times do we do this? Again, pardon me, we do it *infinitely* many times.

The question is this:

How many oranges are left in the basket after we have done this infinitely often?

Anyone care to guess? By the way, if you don't like the process being infinite and then me asking what happens after the *infinite* number of steps, simply do each step at half the time of the previous one. Then if you take one minute for the first step, in 2 minutes you will be able to do this infinitely many times.

No guesses? German scientists are probably shy. So usually, people say, "Infinitely many." People may say, "0." Other people say, "1." I say "17." And in fact the answer is: All the responses are correct! Really! And the answer does not depend on magic. It depends on how you work. I will now show three scenarios.
Here is the first scenario. We take out oranges number 1 and 2, and we put back number 2. We take out 2 and 3, and we put back number 3. We take out 3 and 4 and put back number 4. Doing this infinitely often, what happens? All the oranges are eventually outside of the basket. How do you prove that? Name your orange, say 758? In the 759th step, your orange has been taken out the second time, and it remains out. Hence, all oranges are outside. This means that zero oranges are left inside. Zero!
However, if we work slightly differently the answer will be different. Suppose we start the same way, oranges 1 and 2 out, 2 goes back in. Now instead of taking out 2 and 3, we take out oranges 3 and 4, and we put orange 4 back in. And then we take out 5 and 6, and we put 6 back in, etc.
At the end of the day – this infinitely long day of the second scenario – all the odd-numbered oranges are outside, and all the even-numbered oranges are left inside. So, you are left with infinitely many oranges inside the basket.
Now why is 17 also a good answer? How do I get 17? Very simple, I do 17 steps of the second scenario, and then I continue with the steps of the first scenario. That's it. Magic!

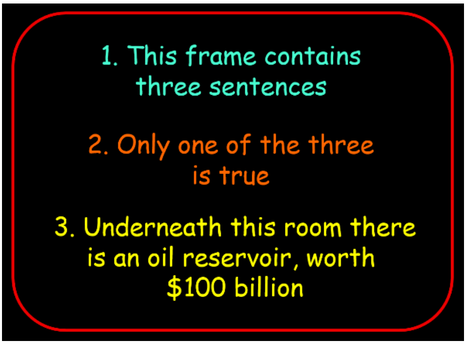
Logic

Logic is really the fundamental of the fundamentals of mathematics, and of rational thought in general. Even though most of you are not mathematicians, logical, rational thinking, reaching a conclusion based on premises, is something that you all do, and we all have to be able to do it well. Mathematical logic has a wealth of extraordinarily interesting things: riddles, paradoxes, and also results, of course. I am going to show you one of each.
The first one is a very famous and cute paradox:

"I am now lying".

Why is this a paradox? Because it cannot be true. If it's true, then it's false because it says that I am lying. It cannot be false because, if it's false, then

it actually says that I am not lying, which means that it is true. So, this is a paradox.
And this paradox is based on the fact that you can't really formalize the notion of "This sentence is not true," or "I am now lying." You cannot formalize this in a system of mathematics with classical means of axioms and rules of inference. So that's why it's a paradox, and people like to amuse each other with it.
In fact, this paradox is so nice that you can dress it up in a kind of Halloween disguise that makes it look a lot more profound. I am going to show you one of the dress-ups of this paradox.
All you see here now is a kind of box, a rounded rectangle. I am going to write things in it. When I write these things, please do not try to convince yourselves as to whether what I have written is true or false. Just make sure that my English is okay and that you agree that each of these sentences makes a clear statement, which is either true or false, but don't try – yet – to get into the issue of whether it actually is true or false.



Sentence number 1: "This frame contains three sentences." English, okay. Makes sense too, so you understand what I am saying. Sentence number 2: "Only one of the three sentences in this frame is true." Again, English okay, the statement that only one of the sentences is true." Forget now about whether you believe it or not.
And, Christoph, this is also for you, sentence number 3: "Underneath this room, there is an oil reservoir worth US-$ 100 billion."
Now, if I can prove this to you, we should stop this lecture immediately; give up on the election process and all these other important things you guys are going to do later, and start digging. Sorry?

MALE VOICE Digging for oil is not allowed in Berlin.

DAVID HAREL So topple the government. We've done that at least once in our country.

Let's go slowly now and ask ourselves about the truth or falsity of the sentences. Is the first one true? Yes, okay. Now let's look at number two. It's either true or false, because you admitted that it makes a clear statement. If it's true, then we already have two true sentences, but it says that only one of them is true. So, it cannot be true. It has to be false.
What is the falsification – the *opposite* – of "only one"? Either zero or two or more. Zero is impossible because the first sentence is already true. So it can't be that zero sentences are true. So there have to be at least two true sentences. But if the first sentence is true and the second is false, the third sentence has to be true!! So again, this is not a theorem. It is a paradox because the sentences are talking about themselves.
That was kind of – not a comic relief about logic – but that was the paradox part of logic. But now let's take a look at the statement "I am now lying" once again. This is a nice little paradox.
It's not that this sentence is false. It simply cannot be formulated using mathematics or using a particular system of mathematics.
But now, dear ladies and gentlemen, let us look at something that is not a paradox at all. In fact, it goes to the very heart of mathematics and human thinking, and in my humble opinion, it is one of the most fundamental and most beautiful results in mathematics ever.
We are talking about a result by the Austrian mathematician Kurt Gödel, proven in 1933:

Any proof system for mathematics will necessarily be incomplete.

That is, there will always be true statements that cannot be proved. Boom! This theorem put an end to attempts and speculations, including those by David Hilbert, who, I am sure, many of you have heard of, at the beginning of the 20th century. Hilbert challenged mathematicians to try to provide a proof system that is rich enough to be able to prove anything that is true, at least in principle. The proofs may be hard. We might not be smart enough to *find* the proofs. However, in principle, concerning any statement that is true, you want to know that there is proof of it within mathematics or specifically within your mathematical system.
People tried and speculated. Then more than 30 years later along came Gödel and showed that this quest is not even worth thinking about. There will always in *any mathematical system* be false statements where we cannot prove they are false and there will always be true statements that we cannot prove to be true.

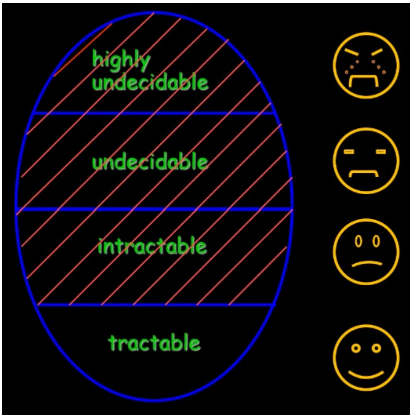
How did he do this? I cannot get into all the details, but we can discuss the heart of the proof. His full proof is really a *tour de force*, an act of utmost brilliance and utmost beauty.
The first thing Gödel did was to replace the notion of, "I am not true," with the notion of,

"I cannot be proved"

Now the main technical part of his proof is to show that, in any sound mathematical system of axioms and rules of inference that is able to talk about (at least) the integers, he can find a devious and very clever way to write down this sentence *within* the system.
In fact, Gödel showed that, in any such mathematical system, he can build a whole contraption of mechanisms to be able to formulate this sentence. And once you've written it down inside the system, what does the sentence mean, "I cannot be proved"? Is it true or false? It cannot be false. Why? Because if it is false, it can be proved, and the system of mathematics is sound. So, it has to be true. It cannot be false. QED! We have just found a sentence that is true, but it states that it cannot be proved.
This beautifully elegant distinction between truth and proof in a system is what Gödel's argument rests on. Again, I am saying that this is one of the most important results in mathematics. It is also a hard result to achieve. However, the most important thing is that it goes to the very heart not only of mathematics but also of human reasoning in general.
No matter how hard you try, you will never be able to find a mechanistic, algorithmic, computerized step-by-step notion of what it means to prove something that's true without it necessarily admitting statements that cannot be proved in your system despite them being true!
So, mathematical proof, human proof, rigorous proof, are all incomplete. And this is not a theorem in topology, it is not a theorem on numbers. Gödel's theorem is a proven, incontestable fact about our very ability to prove things.
To leave you with a little piece of neat homework, logic has, of course, many riddles. There is a very simple and famous riddle, which is the following. You reach a branch in a road, where one branch leads to Berlin, the other to Frankfurt. There is no sign, but there are two people there. You know that one of them always tells the truth and the other always lies. Okay? You want to go to Frankfurt. And you only have one chance at asking one of them a question and getting safely to Frankfurt.

The answer to this riddle is that you point to one of the two people, and you say:

"If I were to ask your friend how to get to Frankfurt, would she say this way?"



And you do the opposite. Think about that. It is easy. If you happen to address the truth-speaking person, you get the opposite answer because he or she says the truth about the liar. If you happen to address the liar, he or she lies about the truth. So, in either case, if you do the *opposite* of the reply you receive, you get to Frankfurt safely. That is the easy riddle.

Now here is the hard one. I will not show you how to solve it. It's a really nice one worth thinking about. This time you have *three* people. It's nothing to do with Frankfurt or Berlin. You have three people. Mr. or Mrs. X always speaks the truth. Mr. or Mrs. Y always tells lies. And Mr. or Mrs. Z is a politician and answers yes or no at random. Not only is the answer given at random, but also you cannot even be sure that if you asked Mr. or Mrs. Z the same question twice, you would get the same answer. It is completely random.

And the issue now is this: Identify the three persons, with only three questions at your disposal. The definition of a question is as follows. You address it to one of the people, it must be a yes-no question, and that's it. Of course, the fact that this riddle looks very difficult, and is indeed a lot more difficult than the previous one, is because, how on earth can you rely on an answer you get if you happen to have addressed the third person here, Mr. or Mrs. Z? Nevertheless, it has a very beautiful solution, which I recommend you try to find.

Algorithmics

Algorithmics, the theory and practice of algorithms, is really the mathematics of computer science. I come from logic and algorithmics. Thus, this area is closer to my heart than some of the others.

I will say a few things about algorithmics. First of all, the world of algorithmic or computational problems – all the way from computing the average salary of the employees of the Berlin-Brandenburg Academy of Sciences and Humanities to trying to predict and calculate the optimal route of an airplane coming into land – these formally defined problems have been divided over the last 50 years by computer scientists into four rough categories: There are the ones that are tractable, so they can be solved with reasonable amounts of resources, computer time, computer memory, and so on.
Then there are the undecidable problems. It can be proved that these problems do not admit any solutions. Just like, you don't go around looking for triangles that have equal angles but not equal sides because there are no such things, and you or someone else can prove it. These problems have been proven not to admit algorithmic solutions on *any* computer, no matter how large.
Above these are things that are even worse than not being decidable, but I won't elaborate on this curious-sounding notion at all.
There is a very interesting class of problems between the tractable and undecidable ones. These problems do have, in principle, algorithmic solutions. You can write programs to solve them, sometimes easy-to-find algorithms, but the amount of time and/or memory that they require is provably unreasonable. Unreasonable does not mean I have to wait 3 hours. It means I have to wait more time than the time that has elapsed since the Big Bang. And as the input to the problem gets larger, things get even worse. And here, a larger and better computer will make only a small difference. For the more mathematically inclined people, I should add that the term "reasonable" means that they can be solved in a polynomial amount of time.
This is just a rough breakup. There are many more subtle and delicate breakups within each of these areas. However, really, the important line is the one between tractable and intractable. Is this a problem I can essentially, in principle and in practice, solve with reasonable resources, or is it not, either not computable at all, i.e., undecidable, or intractable?
What I want to do here is tell you about something that touches upon all of these, which is another brilliant piece of work done – by several researchers – over a number of years. It is called *zero-knowledge proofs* and it is relevant not only to computer science and to mathematics. It is relevant to how we prove things to each other. How do I convince you about something? Let me try to tell you the story of zero-knowledge proofs using two examples that anyone can easily understand. They don't look mathematical at all.

Imagine I tell you that I know a particular secret. You hear what the secret is that I am going to tell you. You are astonished and say, "No way. No way can you know that." And I say, "Yes, yes, I do." You say, "No," and I say, "Yes." You say, "No." And then you say, "Prove it to me."

Just for the sake of the argument here, let's say that I claim that I know the color of the socks Vladimir Putin is wearing at this very moment. What is your response to that? "With all due respect, David, no, you don't." You know that I do not know Putin. You don't even believe that I have a friend who is close to him now. You say, "No, you are lying." You are essentially 100 percent certain that I am lying. And I say, "In fact, I do know." And then you say, "Prove it."

How can we imagine a proof protocol that will convince you beyond any doubt? And you get to define what shade of doubt will satisfy you. What we can think about is this. We turn off the microphones, make sure there are no cameras here, and then I tell you the color, say "light purple." Then we call Putin in, and we ask him to do this or that, so that you can see his socks. And you see light purple. What do you say then? "Sorry, we were wrong; we actually do believe you knew."

Now of course, I could have just guessed, but there are thousands of colors, so the chances of me guessing right are close to nil. You are essentially pretty certain that I was not lying.

All this is fine. However, the problem with this protocol of proof is the following. Not only are you now convinced that I was telling the truth, that I really knew this secret, but now *you* also know the secret! But suppose I do not want to give away the secret itself, but only want to prove to you that I know it. Moreover, I want you to know nothing more after we go through this process than you knew earlier, and also that you can't infer the secret from what I tell you.

If you have connections to the Kremlin, you may be able to find this out, but you don't. And I don't want you to even know if it's a light color or a dark color, or if it's a warm color, like orange or red, or a cold color, such as blue, black, or purple. And for that, you are probably going to say, "Excuse me, sir, no way. There is no way you will be able to convince us that you know a secret unless you put the secret here in front of everyone, and we check."

The surprising fact is, that you can indeed do that, without giving away any information about the secret itself, except to convince you that I indeed know it.

By the way, this kind of zero-knowledge proof has many applications, including many things we do with our credit cards, where we just want to divulge the information that we have enough money in our bank, but we don't want the clerk in the bank to know how much we have. In other words, some things you want to let out; some things you don't.
I am going to actually show you how to do this using a really simple colorful example, which again does not look like it has anything to do with mathematics. That is the game called "Where is Waldo?" How many people are familiar with this? In our country, it's called "Where is Effy?" "Where is Waldo?" is a series of books that you can buy for your grandchildren or children. The books are very large. Every page of such a book shows a very, very, heavily cluttered picture.

In Germany, a picture like this is called a "*Wimmelbild*". And there is a tiny little person called Waldo. I think he has a striped T-shirt and maybe short jeans and a hat. Waldo appears in every one of these pictures somewhere. Sometimes, he is easy to spot, sometimes Waldo is hiding behind a bush, and you just see a little bit of him protruding. And sometimes he is *very* hard to find. It's a wonderful game to play.

Now suppose that you and I are playing this game. We have spread out this picture on the floor. We spend 15 minutes or 20 minutes intensely inspecting it. And suddenly, I say, "I know. I found him." And you say, "No." And I say, "Yes." And you say, "No."

Now can anyone here think of a way, a real actual way, I can convince, say the gentleman over there, that I know where Waldo is and that I have found him, but without divulging anything about his whereabouts; whether he is in this part of the picture, that part of the picture, whether he is behind a bush or a house, and so on? Can you think of a way to do this?

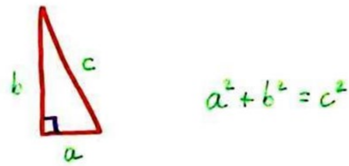
Here is one easy way of doing it. I bring into the room a very large sheet of paper, at least twice as large as the *"Wimmelbild"* picture. You turn your back to me just for a minute, I cut a little hole in this paper, move the picture around on the floor in some random way, and I put the paper over the shifted picture. Then you look at this hole, and you see Waldo.

Looking through this hole, you only see a little bit of Waldo. You have no idea where he is in the page because this is a very large piece of paper. And you're convinced. This is a zero-knowledge proof! When we're done, you have no idea where to find Waldo on the picture, but you are completely convinced that I have just successfully performed "Waldo finding".

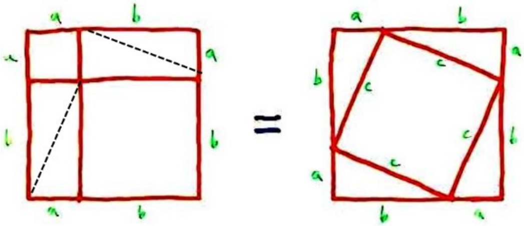
I just want to add here that deep investigations around this notion of zero-knowledge proof have been carried out in the last 40 or so years of brilliant work by many computer scientists. Among other things, one has to define the mathematical notion of a secret. This has to be something that you don't believe I can find out easily. What does easy mean? More precisely defined, easy means algorithmically tractable as introduced before; namely, doable in polynomial time.

Concerning the secret about Putin, I can find out the color of Putin's socks by driving up, or flying to the Kremlin, or by bribing someone. These are big efforts. To make this mathematically sound, you need a mathematical definition of a secret, you need a mathematical definition of what it means to know something, you need a mathematical definition of proof; and of course, you need a mathematical definition of zero-knowledge proof, which is what I just tried to explain. All this has been achieved, and there are many, many beautiful zero-knowledge techniques for a wide variety of problems (secrets). There are also a whole slew of results concerning the kinds of problems that in principle admit zero-knowledge proof vs. those that do not.

Geometry



I am almost finished. Geometry now. I have one slide on geometry. I could have spent the whole hour talking about geometry. To end my talk, I want to give you a very beautiful, but extremely simple, proof of one of the most well-known theorems of geometry, the Pythagorean theorem, which I am sure you are all familiar with. In any triangle with a right angle, $a^2 + b^2 = c^2$. In words, the square whose side is the hypotenuse (the side opposite the right angle) is equal in area to the sum of the areas of the squares on the other two sides. It may be that you remember how this was proved to you in elementary or in high school. It is rather complicated. Here is a very simple and elegant proof-by-picture.



What you do is you form two identical squares of size a+b by a+b. The sides of the left square are labelled ab, ab, ba, ba. The square on the right has the same sides, but they are labelled ab, ab, ab, ab. The area of the two squares is exactly the same, of course, let's call it A. This area A is a+b times a+b. Then you draw the red lines shown in the picture in the interior of the two squares.

We now draw the two dashed black lines into the two rectangles of the left-hand square, turning each of the rectangles into two triangles. These four triangles have identical area, as you can see immediately. Four triangles of the same area also appear in the right-hand square. If we subtract the sum of the areas of the four triangles from the area A of the left-hand square, what remains is the sum $a^2 + b^2$ of the areas of the two small squares shown inside the left-hand square. And if we subtract the sum of the areas of the four triangles from the area A in the *right-hand* square, c^2 remains, the area of the smaller square inside the right-hand square. It follows that $a^2 + b^2$ has to be equal to c^2. End of proof, QED, beautiful!

Panel

Beauty as a goal and as a value: Five perspectives from the Academy

GÜNTER M. ZIEGLER Dear David, thank you very much for a beautiful lecture with so many different things from mathematics and computer science and, whatever it was, politics. I would propose that we do not add a discussion here now, but that we go into the second part of this that we have prepared, namely, a sequence of five impulses.
In that case, I was even more stricter in terms of putting limitations on the speakers. We have five small presentations with a maximum of five minutes each. I would propose we do them in the order that was announced in the program.
So, all of them will talk about beauty as a goal, as a value, as whatever it's good for. Ute Frevert will start and, I guess, talk about emotions. Then Horst Bredekamp will talk about images. Mike Schlaich will talk about buildings, Julia Fischer about evolution, and Christoph Markschies about writing history. Your five minutes start now, Ute. Welcome.

Ute Frevert | On gender history

I will start with the following line that provides a perfect transition from David's talk: "My father was Professor of Mathematics at Munich University, and my mother was a very beautiful woman."
This is how Katia Mann, the wife of Thomas, begins her *Unwritten Memoirs* published in 1974. Beauty or the lack of it enters those memoirs whenever women are concerned. Katia never comments on the beauty of mathematics, although she studied it at university. Yet she talks, repeatedly, about her mother as a famously beautiful woman.
She dryly mentions that her own prettiness paled alongside her mother's beauty. This mirrors the situation in Thomas Mann's family of origin. His mother Julia was known as the most beautiful woman in town, meaning Lübeck. Thomas

literally worshiped her and admired her ivory *teint*, her classic nose, and the most adorable mouth that he had ever seen.
His sisters, however, suffered, as Julia's beauty continued to attract suitors after her husband's death. According to Katia, men always hovered between courting mother or daughter.
For Katia, beauty said it all. In her autobiography, men were presented at length in terms of their professional roles, their musical tastes, or in terms of whom they knew and conversed with. Women, in contrast, were looked at through the lens of beauty, the beauty of their faces above all but also the beauty of their physical shape.
Katia could have told long stories about her mother Hedwig Pringsheim, who had worked as an actress before marrying the Professor of Mathematics, and who had many more talents than just being beautiful, talents that were widely appreciated in her social circles.
Julia Mann was likewise admired for her exquisite piano performance. Her daughter-in-law, however, put it this way: "Sie spielte ganz hübsch Klavier und sang […] Außerdem war sie auch zeichnerisch nicht ganz unbegabt."
It is hard to miss the tone of condescension and belittlement – a belittlement that Katia also applied to herself. Although she was the granddaughter of a feminist pioneer, she saw herself as the companion of a very important man.
This self-concept informed how she judged other women. As long as they were beautiful, they mattered. A lack of beauty condemned them to invisibility and insignificance.
Katia Mann was not the only one to make such judgments. You could hear and read them all over the place in what was called 'good European society' during the 19th and most of the 20th century.
Beauty came in many variations. The exotic beauty of Julia Mann, who had a very beautiful Brazilian mother, differed from the Central European beauty of Hedwig Pringsheim. *La belle juive* was another type that captured people's imagination.
Unfortunately, I cannot go into the details and images here, but I can offer a brief historical glimpse that starts, of course, with the mythical judgment of Paris, who was commissioned to choose the fairest of three Greek goddesses. Yet, Greek culture was far from worshipping female beauty only, as male sculptures tell us, and as Renaissance artists rediscovered.

In contrast, medieval troubadours focused on singing the praise of noble and beautiful women. In their view, beauty, noble birth, and highest virtue, went together, but this was soon contested.
Take the Grimm fairy tale Snow White. Every morning, the wicked but beautiful queen consults her mirror, “Magic mirror on the wall, who is the fairest one of all?” When the truthful mirror eventually acknowledges that Snow White’s fairness surpasses that of her stepmother, the latter takes violent action.
During the age of industrial capitalism, beauty was not only framed as the monopoly of women. Beauty also turned into a marketable good in several ways.
First, it was considered an asset on the marriage market and an instrument of women’s upward mobility. Second, it was the target of an expanding beauty industry that, up to this very day, has developed an endless number of beautifying products, from cosmetics to surgery, from fashion to diets and gymnastics. Consumers have been overwhelmingly female. Only very recently, have men started to catch up, with gay men taking the lead.
Thirdly and closely connected to the commodification of beauty, mass media have become powerful agents in visually producing and disseminating beauty standards. This goes hand in hand with beauty pageants, which were invented in the mid-19th century.
In 1921, Atlantic City staged the first Miss America contest, and the practice quickly traveled to Europe. It was enthusiastically covered by the press and established deep ties to commercial culture. It also served to both democratize and streamline concepts of physical beauty across social classes and ethnic groups.
Such streamlining – and this is my final point – has now increasingly come under criticism. What is considered beautiful is open once again to a wider range of viewpoints and interpretations. Fashion and cosmetics models may have curves nowadays, can be people of color, or even have disabilities. They have also stopped being young and exclusively female. In turn, women are no longer, as in Katia Mann’s memoirs, solely assessed in terms of beauty.
Yet beauty still matters. People tend to pay more positive attention to physically attractive people, whether they are male, female, or non-binary. Gender differences also persist, due to the gendered character of power. A rich and old but short and ugly man can easily find favor with beautiful women. The opposite has rarely been reported on. While there is no lack of unattractive women, they are usually not in a position to exert power and thus gain the attention of attractive men.

I would love to go deeper into this triangle of beauty, power, and gender and explore its historical trajectories, but my time is over. I now hand over to Art Historian Horst Bredekamp.

Horst Bredekamp | On images

My topic is mathematics, and art and the concept of beauty. It might be, in a sense, a comment on the point David Harel omitted.

Morphology teaches that any form of design in nature as well as in art is interwoven with the phenomenon of beauty. The concept of beauty of man has its starting point in his own body, in order to be able to recognize a structural principle of the universe from this determination. Objections to this connection between microcosm and macrocosm have been raised repeatedly, but no less frequently rejected in theology, philosophy, the natural sciences, and art. The likeness of man to God was the biblical variant to affirm this approach. The second was the ancient aesthetics of proportion.

Vitruvius stated that, if man stretched out his arms and legs, he could be placed in a circle and a square with the navel as the center. This statement had been related to the cosmos again and again, and preeminently by the widely read Agrippa von Nettesheim, who posited that there is no part of the human body that does not correspond to a celestial sign. Thus, mathematics served the idea that there is a connection between the nature of man in his ideal form and the universe.

Among the most impressive visualizations of this assumption is one of the visions of Hildegard of Bingen, as in the Lucca illumination of 1230. A human being transcends the earth, reaching out with his slightly outstretched arms in order to touch the inner circle of space. However, von Bingen did not envision the center in the navel, as Vitruvius had stated, but in the sex. Deviations of this kind pervade all visualizations of Vitruvius' definition of the ideal human being.

There was a connection that could not be better expressed than in the concept of beauty assisted by circle and square. Leonardo's Vitruvian Man, however, distinguished between the navel as the center of the circle and the sex as the center of the square. Only those, who like Cesare Cesariano in 1521, exposed people to the rack could attempt to realize Vitruvius's specification in actuality. It was a fiction.

Because of these irritations, Albrecht Dürer, who had made numerous studies of proportion during his trip to Italy in 1505-1507, undertook the most elaborate search for the canon of beauty to date. Recent findings by the editor of his authoritative work *Von menschlicher Proportion*, Berthold Hinz, permit a new view of his efforts. In his studies of proportions, Dürer took out any form of movement in order to obtain a flat representation on which measurements could be taken and compared to the proportions. In this way, he was able to transfer the aesthetics of proportion into anthropomorphism and thus prescribe that diagrammatic form which was rediscovered in the industrial age and which still conditions all ergonomic-metric considerations.
He made more than 200 measurements with variable sizes, up to very overweight and excessively slender men and women, with a view to the ideal measurements of a mathematically hypothesized ideal beauty that did not yield any normative result.
Looking at the ideal measurements of a mathematically hypostasized ideal beauty, these measurements yielded no result. Dürer's figures are not beautiful, and they were considered aberrant even to many of his contemporaries. It was not understood that they offered the only possibility to show in their rigid frontality that a concept of beauty cannot be assumed, but rather the enjoyment of variability and diversity.
Perhaps the most beautiful sentence, which ever came from Dürer is accordingly, "But what beauty is, that I do not know." Dürer added a so-called twin to his figures, which transferred the measurements into a coordinate system. In this way, Dürer anticipated analytical geometry 150 years before René Descartes.
To the right of the twin appears, not without reason, a serpentine line that looks like a treble clef. It was considered the incarnation of diversity and free imagination. It also adorns the title page to indicate the character of a beauty that can only be understood as movement and the interplay of symmetry and deviation.
In the 18th century, William Hogarth defined this serpentine line as the epitome of creative forms of beauty in art and nature. *Analysis of Beauty* is the title of his epochal work in whose pyramid of eternity the serpentine line appears – variety as the condition of beauty. Through Hogarth, the hope of finding the same concept of beauty in both art and nature was once again proposed.
Charles Darwin took on Hogarth's definition of beauty as variety in order to be able to describe the mutations, which were based on sexual selection. Long

marginalized, Darwin's conviction has come back with a vengeance, most recently in the publications by Richard O. Prum and Christiane Nüsslein-Volhard. Repeatedly declared dead, the sense of beauty has returned as a universal principle. Originating from mathematical measurements, its condition is now based on the interplay between norm and deviation. Beauty aims at variation. Without a minimum of disturbance, true beauty cannot be achieved.

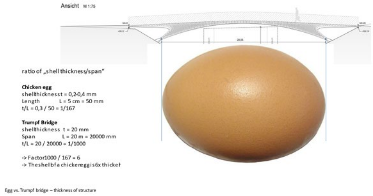
Mike Schlaich | On buildings

Good afternoon! My name is Mike Schlaich.
I will talk about beauty in engineering and in particular about beauty and engineering in structural engineering. I would like to explain it to you using the example of a bridge.
For me, beauty in bridge engineering has to do with elegance, which I define as effortless beauty. It has to do with light structures that use a minimum of materials and resources. Moreover, it has to do with visibility, with the visible flow of forces. Because we like what we understand. How does a structure work?
I will show you a very light bridge that describes the flow of forces in the best possible way. I also want to talk about beauty in the process and in the product.

© sbp/ Andreas Schnubel

In civil and structural engineering, we like shell structures because they are very efficient. It is a very efficient way to carry loads, through double curvature.
The bridge we designed to be very slim. It is 30 meters long but only 20 millimeters thick, six times thinner than an eggshell in proportion.

© HG Esch Photography

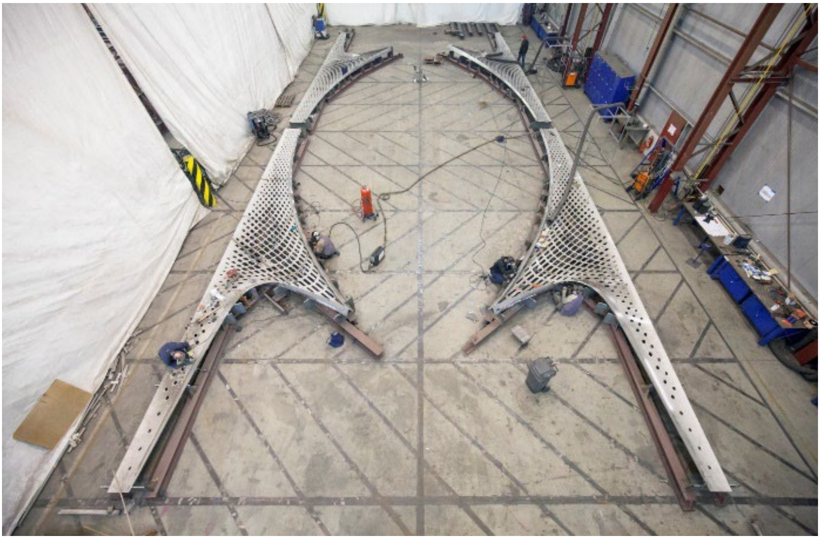


© sbp

You can show the flow of forces, the flow through a wall on the left on this slide or the flow of forces in a shell structure, in a cylinder, by showing the principal stresses or the principal membrane forces, but you can also build models. You can go to the supermarket, buy 2 kilos of oranges and stretch the net. When you stretch the net, you come to a shape that shows the flow of forces. And it has a lot of holes in it.

© sbp/ Mike Schlaich

You can also calculate the flow of forces. Now we move on to the design process of this bridge. Today there are wonderful tools that allow you to calculate things that you could not calculate years ago.
The bridge connects two areas of a factory. The owner of the bridge is Trumpf Laser Cutting. You can take a 3-meter by 6-meter stainless steel plate and cut out all the holes that you've designed, thanks to all these wonderful software tools that we have as engineers today.
But still, it's plain. We want to bend it, to have an eggshell, so to speak.
After laser cutting, we now go back in history, to a shipyard. There is this gentleman who bends steel the way it has always been bent, and there is a film, but I unfortunately don't have time to show it. Basically, he plastically deforms the steel so that you get the ship shape using simple equipment and the highest level of craftsmanship and traditional knowledge.


© wilfried-dechau.de

Then everything is assembled. Prefabrication is very important here. I think it is also a very elegant way of building without producing a lot of waste. If you look closely, you can see the edges of this bridge, which, like a collar, are for additional stiffness.

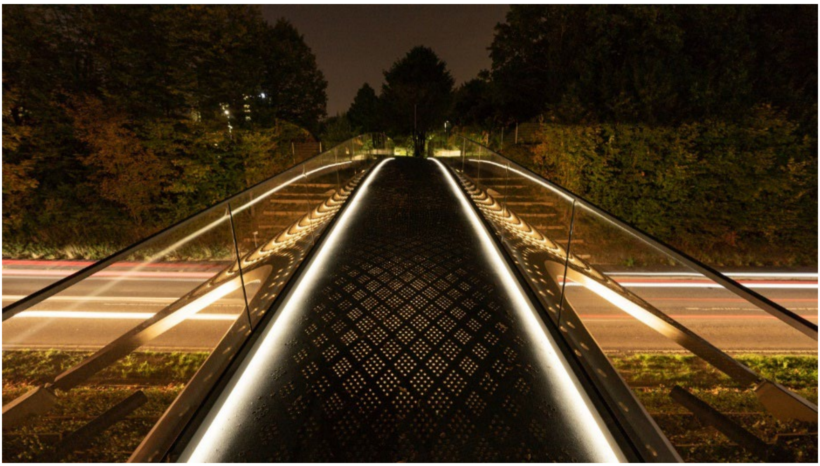
Three edges of the shell need to be stiffened. And that's what you learn when you study the works of famous builders, that is, when you learn from history. This is a famous concrete shell in Spain, and you see that the edges are bent upwards to stiffen it, so we can use this idea for our bridge.

Then it is prefabricated, and you use the prefabricated elements and assemble the entire thing on site in a tent. You take the tent down again and the crane comes. It's only 20 tons of steel. A crane can carry it. You can lift it, but if you lift it like a piece of paper, it will deform too much. So you need an auxiliary structure, and you must design that. Then you put it in its final position, but you must support it so it can function like a shell. The support must carry the load but allow for rotation. We need hinges.

Again, we learn from history. Look at earlier shells and you see how they were supported. There is a hinge, just a spherical ball. So why not learn from history? Bad artists copy, good artists steal. That's why we transferred the hinge to this little bridge. The details must be carefully designed, and this adds to the beauty, I think.

© wilfried-dechau.de

The bridge is supported like the orange net, and you can see all the holes. You see the flow of the forces. You understand how it works. But you could disfigure the whole bridge with a heavy handrail with railings. We found museum glass that is non-reflective and very transparent, and suddenly you can hardly see the handrail. You can even drill holes in the walkway that are not as big as



© sbp/ Andreas Schnubel

the holes outside the walkway, but you can still see the flow of force if you drill properly. With computerized CNC machining, it's not a problem.
You have to illuminate the bridge so that it also looks nice at night, because our structures exist at night half of the time. Therefore, lighting is very important, and it adds to what I would call beauty.
The combination of functionality, aesthetics and technology is what makes the bridge special. The bridge is also the first of its kind where the shell is both the supporting structure and the walkway. What I particularly like is the combination of old traditions and very simple methods with high-tech approaches.
When I speak of elegance as effortless beauty, we find that something is elegant when we feel but do not see any more all the work that was needed to achieve a structure. It appears natural.
This is the last picture. For me, beauty in buildings is a must, and if a building does not look good, it should not be built.
Thank you.

Julia Fischer | On nature/evolution

"On the whole, birds appear to be the most aesthetic of all animals, except for man of course, and they have nearly the same taste for the beautiful as we have." That is what Charles Darwin wrote in his famous work *The Descent of Man*, *and Selection in Relation to Sex,* published in 1871. And in this way, he made the question of beauty an issue for evolutionary biologists.
Initially, when he started thinking about evolution, he was very much occupied by the question as to how animals adapt to the environment and how they respond to selective pressures, such as not being detected by anyone, irrespective of whether you are prey or predators.
But then there were some animals whose appearance greatly puzzled him, such as the birds of paradise, where males sport elongated feathers that may look beautiful but don't favor a bird's survival. In 1860, a year after the publication of his work *The Origin of Species*, he thus wrote, "The sight of a feather in a peacock's tail whenever I gaze at it makes me sick." Luckily, it did not only make him sick, but also forced him to think about the role of sexual selection, according to which some traits that an animal possesses are a result of mate choice and mate selection by the opposite sex.

And we are not just talking about ornaments in the form of feathers, hair or coloration, but also behavioral displays such as the birdsong. At least in temperate regions, it's the male who sings to attract females, but also to repel male competitors. Thus, there are different ways in which ornamentation, either visual or auditory, plays a role in sexual selection. Ornaments can also encompass buildings. Bowerbirds, for instance, produce wonderful constructions, and they put ornaments on and around them, including glass shards, to attract the opposite sex. In birds, it's typically the males that look more beautiful and have evolved ornaments in response to female preferences. And we know now that these female preferences play a huge role in bringing about the most conspicuous and wonderful forms in life. But we also see that females have evolved traits to attract males. And in the case of the sexual swellings of non-human primates, one can clearly say that beauty is in the eye of the beholder. We can generally group such sexually selected traits into two major classes. Some are index signals that directly reflect the health and stamina of the individual, while other signals are 'costly' signals.

They either require energy to produce or afford some indirect cost, such as a higher probability of being targeted by a predator. While the costs must not be very high in absolute terms, they must be relatively more costly for an animal in poor condition to produce. Thus, when you see deer with impressive antlers, it is clear that only males in good condition can bear the costs. Therefore, these signals are also known as handicap signals. It was Amotz Zahavi, a famous Israeli scientist, who developed the concept of the handicap signal. And it was then the mathematician Alan Grafen who provided the proof that this theory was correct. The question of beauty from an evolutionary perspective is a very interdisciplinary one.

Finally, I would like to add, that if you look at mate choice in animals, it's not just the looks that are decisive. Mate choice can also be driven by the assessment of another animal's kindness or the willingness to provide care for the young. Thus, it's not just appearance but also inner beauty that plays a role in mate choice. Thank you.

Christoph Markschies | On writing history

Does beauty play a role in the historical sciences? Or to ask more precisely, is beauty an ideal when historians publish the results of their research, when they write history? The Deutsche Akademie für Sprache und Dichtung in Darmstadt (whose President is currently our member Ernst Osterkamp) awards its Sigmund Freud Prize for Academic Prose – which went for example in 2020 to our member Ute Frevert – not for beautiful writing but to scientists, and I quote here, "who publish in German and contribute decisively to the development of language usage in their field through an outstanding linguistic style."[6]

The Anna Krüger Foundation Prize at the Wissenschaftskolleg zu Berlin, with four members of the Academy among its nine prizewinners, is also awarded not for beautiful writing but to a scientist who has, I quote, "written an outstanding work in a good and comprehensible scientific language."[7] Again, it is not exactly about beauty. At first glance, skepticism is called for when answering the question of beauty as an ideal of historiography. That was once quite different.

Classical premodern and early modern historiography at any rate followed the rules of the classical antique rhetoric in its vast majority of genres.[8] As a matter of course, with its orientation towards *the virtus bene scribendi*. Beauty, however, is not a rhetorical category in the strict sense. Beauty, τὸ καλόν or τὸ κάλλος resp. *pulchritudo* or *pulcherum,* can be said first of all of the object, but then also of the manner in which a text is written, and finally of the ethical attitude of the one who writes. Beauty thus certainly plays a role in classical historiography as a value of historiographical work.

The manner, in which writing is done on the other hand, demonstrates rhetorical adornment (*ornatus*). That is a pleasure-inducing power (*suavitas,* "agreeableness", *dulcitudo*, "compatibility*", iucunditas*, "stimulus"), colorfulness (χρῶμα, *color rhetoricus*), and euphony. This pleasure-inducing power can show itself in the charm and grace (*venustas*) of a well-designed text, but also in passionate vehemence. The desired beautiful linguistic form of a text *(pulchritudo verborum)* is also safeguarded by rules to avoid exaggerated ornamentation, false splendor (*cincinnus*, "turned", *fucus*, "false") or the lack of balance in the linguistic structure (*deformitas* as the opposite term to *pulchritudo*). However, the "beauty" of the subject matter, a "beautiful" ethical attitude of the one who writes about it, and the aforementioned criteria of an embellished, well-formed text are closely connected, and to these two dimensions of ἦθος and

---

6 https://www.deutscheakademie.de/en/awards/sigmund-freud-preis (last accessed on 11/25/2022).

7 https://www.wiko-berlin.de/en/institute/initiatives-cooperations/anna-krueger-foundation (last accessed on 11/25/2023).

8 Anne Eusterschulte, "Schönheit, das Schöne", in: Historisches Wörterbuch der Rhetorik, vol. X, Darmstadt 2013, 1142-1193.

πάθος have been added, since Aristotle, the reasonableness of speech as a third dimension: λόγος, "reason". Accordingly, exposition is characterized not only by a well-ordered sequence of sentences and thoughts (εὐσχημοσύνη, "elegance", *concinnitas*, "harmonic balance", *convenientia*, "harmonious agreement"), which is characterized by tonality, rhythmizing and euphony as well as the stylistic means of an exciting, varied structure (*varietas*, "variety", *copia*, "fullness", *distinctio*, "incision", *ubertas*, "richness", etc.), but by inartificial rational evidence such as reports by eye-witnesses or documents.[9] However, beauty was never the highest value of historiography: *Historia*, a specific form of narration of history, according to the late antique encyclopedist Isidore of Seville, claims the highest truth content (*nam historiae sunt res uerae quae factae sunt*), higher than the argumentum, the narrative, (*non verum sed verisimile*) and clearly higher than *fabula* (*neque verum neque verisimile*).[10]
Almost all basic assumptions of such a theory of beautiful texts based on classical rhetoric have become problematic in modern times. The self-evident triad of ἦθος, πάθος and λόγος became problematic, the close, even self-evident connection between a well-designed harmonic textual structure and a representation's claim to truth broke down, and truth was replaced by the probability previously assigned to poetics, the *verisimile*.[11] "Beauty" became increasingly suspect as an ideal. Due to time constraints, I cannot, of course, go through the whole theoretical discussion of modern historiography and its forming neighboring disciplines here (that would be a topic for an erudite essay or a clever monograph). However, a quotation from the famous "Historik" lecture by Johann Gustav Droysen from 1857 may stand as only one of many examples: Droysen's understanding of "Schönrednerei" as an aberration of good historiography:
This degeneration of historiography (sc. As in Hellenism, C.M.) is all the more serious because the addiction to popularity, i.e., the depiction not for the sake

---

9 Melanie Möller, Rhetorik zur Einführung, Hamburg 2022, 24.

10 Isidor, Etymologiae I 44,5 (SCBO I Lindsay; further proofs in Reinhart Koselleck, art. "Geschichte, Historie", in: Basic Historical Definitions. Historical Lexicon on Political-Social Language in Germany, vol. 2, Stuttgart 1975, [593-717] 620, n. 130): Nam historiae sunt res uerae quae factae sunt; argumenta sunt quae etsi facta non sunt, fieri tamen possunt; fabulae uero sunt quae nec factae sunt nec fieri possunt, quia contra naturam sunt. Cf. also John of Salisbury, Policraticus II 19 (CChr.CM 118, 112,32f. Keats-Rohan: Tanto que longius a scientia ueritatis aberrant, quanto ad eam tumidius irrumpere moliuntur) and Peter von Moos, Geschichte als Topik: Das rhetorische Exemplum von der Antike zur Neuzeit und die historiae im "Policraticus" John of Salisbury, Ordo 2, Hildesheim/Zürich/New York 21996, 147-153 (historia). 208-237 ("historical truth") and Ranulf Higden, Polychronicon II 18 (II, 372-378 Babington).

11 Koselleck, art. "Geschichte, Historie", 638.

of the matter, not even for the sake of a view, a tendency, but in order to please, and according to the taste of the crowd, comes into practice. Greece has been afflicted with this addiction since the schools of the rhetors. This unfortunate oratorical trait was all too soon transplanted to Rome and quickly poisoned serious history. The further down it goes, the more unbearable it becomes – I would remind you of Ammian Marcellin – until it becomes a formal labelled historiography in the Byzantines and sinks into servilism and tastelessness".[12]

And a second example, more closely linked to our Berlin-Brandenburg Academy of Sciences and Humanities, our first President after the reconstitution, the biologist Hubert Markl, quoted in an essay with the provocative title "Top-Level Research Speaks English" (provocative especially for humanities scholars) a sentence by Munich Romance scholar Karl Vossler from 1923. I quote Vossler and Markl, "According to the view we [Vossler and Markl] hold, however, accuracy" – Genauigkeit – "would be precisely the own and special beauty of scientific prose" – "Genauigkeit ist die eigentliche Schönheit".[13] This sentence perhaps most clearly marks the disappearance of the classical ideal of beauty from scientific prose and thus also from the narrative of history.

In the end, even the telling of history became problematic in the twentieth century. Jürgen Kocka, who has done a lot of written work in this field, could tell us about this in detail if he could be present here today. "Don't tell, analyze" was the succinct and, in its brevity, naturally simplistic summary of the corresponding critical stance, which wanted to see "a theoretically enriched, analytically proceeding historical science focused on structures and processes" placed at the center of historiographical work.[14] If one takes the first volume of Hans-Ulrich Wehler's *Deutsche Gesellschaftsgeschichte* (German Social History), which deals with the long eighteenth century and was first published in 1987, after its theoretical preliminary explanations ("Allgemeine Strukturbedingungen und Entwicklungsprozesse" [General Structural Conditions and Developmental Processes]), one will not want to attest to the fact that the classical rhetorical ideals of a pleasure-inducing power, colorfulness and euphony were

12 Johann Gustav Droysen, "Historik. Rekonstruktion der ersten vollständigen Fassung der Vorlesungen" (1857), Grundriß der Historik in der ersten handschriftlichen (1857/1858) und in der letzten gedruckten Fassung. Textausgabe von Peter Leyh, Stuttgart 1977, 97.

13 Karl Vossler, "Die Grenzen der Sprachsoziologie", in: id., Gesammelte Aufsätze zur Sprachphilosophie. München 1923, (210-260) 231.

14 Jürgen Kocka, "Zurück zur Erzählung? Plädoyer für historische Argumentation", in: Geschichte und Gesellschaft 10 (1984), (395-408) 397.

at the center of the work.[15] But this direction, too, fell prey to criticism: our member Jürgen Kocka was therefore able to headline a relevant essay on the debates in 1984 with the title "Back to Narrative?";[16] at the beginning he refers to an aborted project of the "Frankfurter Allgemeine Zeitung" to publish a series of historical short stories in which "it was planned to tell one event each ... in the most vivid way imaginable, in such a way that not only the event itself but also a piece of structural history would be conveyed to a wider audience".[17] The idea, which was obviously also oriented towards the ideal of the classical *virtus bene scribendi*, was abandoned after a few months because an insufficient number of contributions were received in Frankfurt that fulfilled the criteria.[18] Our members Christian Meier and Fritz Stern presented sweeping historical monographs on great personalities and were perceived by some colleagues as the spearhead of a neo-historical turn back to narrative.[19] At the time, Kocka warned against reconciling the opposition between theory and narrative too quickly by radically expanding the concept of "narrative".[20] Its dual, introduced not in the sense of strict opposition but for purposes of analysis, is: narrative and argumentation. And Kocka immediately concedes that there are, of course, narrative elements in primarily argumentative structural and process-historical texts. He could have drawn attention to the fact that, according to classical rhetoric, historiographical narrative is also based on argumentative structures. In the end, Kocka warned against falling back behind the discoveries of the second half of the twentieth century into new one-sidedness and theory-less, only seemingly experience-oriented narratives. But he also warned against "shadow boxing" between the alternatives of "argument" versus "narrative".[21]

---

15 Hans-Ulrich Wehler, Deutsche Gesellschaftsgeschichte, vol. 1: Vom Feudalismus des Alten Reiches bis zur Defensiven Modernisierung der Reformära 1700-1815, München [3]1996, 35-43.

16 Kocka, Zurück zur Erzählung?, 395-408.

17 Ibid., 395.

18 One should actually speak here of the debate about Hayden White: Id., "Tropics of Discourse: Essays in Cultural Criticism", Baltimore 1978 = "Auch Klio dichtet oder die Fiktion des Faktischen. Studien zur Tropologie des historischen Diskurses", introduction by Reinhart Koselleck, from the American by Brigitte Brinkmann-Siepmann and Thomas Siepmann, Stuttgart 1986 and id., "Metahistory: The Historical Imagination in Nineteenth Century Europe", Baltimore 1973 = Metahistory. "Die historische Einbildungskraft im 19. Jahrhundert in Europa", translated by Peter Kohlhaas, Frankfurt am Main 1991.

19 In radical selection: Christian Meier, Caesar, Berlin 1982 and Fritz Stern, Gold und Eisen. Bismarck und sein Bankier Bleichröder, Berlin/Frankfurt am Main 1978.

20 Kocka, Zurück zur Erzählung?, 398.

21 Ibid., 408.

What remains? At first glance, only the substitution of the classical ideal of beauty – as in the case of Karl Vossler: "According to the view we hold, however, accuracy would be precisely the unique and special beauty of scientific prose".[22] But doesn't the old, classical catalogue of criteria of beautiful texts come through again under the substitute terms? I recall the two prizes that (like the Sigmund Freud Prize) want to honor an "outstanding linguistic style"[23] or are dedicated to "a good and comprehensible scientific language"[24]. Didn't the ancient world know the word combination καλὸς κἀγαθός, "beautiful and good"? ἐξηιρημένως or *egregius*, was fondly used in connection with beauty. Is "beauty-ness", then, a hidden ideal of historiographical writing? One could almost think so.

GÜNTER M. ZIEGLER Thanks to all of you for these beautiful presentations. We've seen from David Harel's lecture that beauty in mathematics is the beauty of ideas, and beauty is the only test.
We've learned from Ute Frevert that this is problematic and that, as soon as people talk about beauty, they mean something else.
We've seen from Horst Bredekamp that beauty is in the variety and that this is to be seen in Dürer's writings and monograph.
Mike Schlaich has reminded us that – I am challenged to say – even the engineers point to learning from history to capture beauty, but he also claimed that beauty is a must in design.
Julia Fischer has pointed us towards inner beauty as a very important and decisive component.
And Christoph Markschies has claimed various things, but he's also claimed that or quoted that precision is an inherent component of beauty, which again as a mathematician I would definitely understand and put into my framework.
But this is not about my framework. I would just like to invite you to discuss, to respond to the things we have missed in these six different perspectives on beauty in everything we do, and to not name specific sciences or disciplines. The discussion is open.

---

22 Karl Vossler, "Die Grenzen der Sprachsoziologie", in: Id., Gesammelte Aufsätze zur Sprachphilosophie. München 1923, (210-260) 231.

23 https://www.deutscheakademie.de/de/auszeichnungen/sigmund-freud-preis (last accessed on 11/25/2022).

24 https://www.wiko-berlin.de/institution/initiativen-kooperationen/anna-krueger-stiftung (last accessed on 11/25/2022).

Discussion with the audience, whether beauty is, and should be, a criterion …?

HAROLD JAMES A question for David Harel: I've been told by mathematicians that the elegance of a proof is an important criterion. And elegance seems to me also to be an aesthetic category. Would you be prepared to comment on that?

DAVID HAREL Elegance in the proof is really what I meant by the beauty of ideas. So the elegance of Gödel's proof of incompleteness, it's really very, very beautiful and elegant in the sense that the notion of saying, "I cannot be proved," and then showing that you can say this in any system of mathematics is, in my humble opinion, as elegant as anything.
And sometimes, the elegance is in the flow of ideas from one to the other. Sometimes, it's sheer power. Fermat's Last Theorem was proved by Andrew Wiles. The power of that was in the connection between various areas of mathematics if you understand these areas and the elegance in which one of them is interwoven with another and leads to a result.
So, I think the short answer to your question is, when I talked about the beauty of ideas in mathematics, it's the elegance of the argument in the proof, which is a major component of that.

GÜNTER M. ZIEGLER If you look at Gödel's original 1931[25] paper, it is a short paper. Is the elegance visible from how it's presented in the style, or is it only these monumental ideas and insights that are contained in that?

DAVID HAREL Are you asking me?

GÜNTER M. ZIEGLER I am asking you.

---

[25] Kurt Gödel: "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I". In: Monatshefte für Mathematik und Physik. Akademische Verlagsgesellschaft, Leipzig 38.1931, pp. 173-198.

DAVID HAREL I cannot read Kurt Gödel's original paper because I don't understand the language, but I can share with you that it took me about two or three years after first learning of this theorem, and I think it was in my second year computability course, before I suddenly realized how beautiful the idea was and how elegant it was.
And so, yes, sometimes things that are not presented very nicely do still have inner beauty. So, it's two sides of the same coin.

CAROLA LENTZ I would like to throw up the question of the universality of the standards of beauty. Mathematics sounds like it is universal, but coming back to Ute Frevert's thoughts, I would like to take the example of beauty pageants in Africa, which always include a moment of morality as part of what is regarded as the beauty of a woman. So, here we find a highly moralized ideal of female beauty.
And my question to Julia Fischer would be: do you think that different species have different ideas about beauty? This is a hypothetical question, but still, it is a question about the relativity of standards, and how specific do they get? And who judges, and who then finds companions to also share the judgment that something is or is not beautiful?

JULIA FISCHER I don't know whether the concept of beauty is a good one to describe what animals are doing or thinking, but I can tell you for sure that they have differential preferences. So, if we equate a preference for a certain appearance with a sense of beauty, then yes, of course, we may apply that concept. A female nightingale would prefer a male nightingale singing compared to a zebra finch – and I would say every bird should prefer the nightingale's song to that of the zebra finch, but of course the female zebra finches don't. To me, the nightingale's song is the most beautiful bird song there is, but the zebra finch ladies don't like it. What I am trying to say is that there is an evolved component to what the preferences are. This component is very strong. But then there's also an acquired taste. That's also true for animals. We know this from Lorenz' work on imprinting, where early exposure to certain stimuli affects what you prefer later on. And in some animal species, these effects are stronger, and in other animal species, less strong.
What is actually underrated in humans is the importance of experience in our preferences. We tend to find things beautiful that are the average of what we see around us. You can do simple experiments where you have an average

face and you show it to people, and they rate it. And then you gradually but very slowly put the eyes further to the outside. And so people see these faces where the eyes are very far apart, and they come to think that that's beautiful, because now that's the new average. And then you show them the original face, and they think that the eyes are now too narrow. In summary, there are three levels, the evolved component, the acquired component early in life, and then there is also what we currently see and what affects our judgment. And I think that similar processes can also be found in other animal species.

GÜNTER M. ZIEGLER Ute Frevert perhaps wants to comment?

UTE FREVERT You're absolutely right in pointing out that the concept of beauty as it has been applied and is still being applied is a very changeable one. But I do see, especially in the modern era, very little moralizing about it. Beauty, as it stands now, is rarely connected to morals and virtues, as in former times. It is, however, strongly connected to racial – and even racist – ideas, in most parts of the world. What counts as beautiful differs considerably. Power differentials have a say on whose standards of beauty carry more valence than others. Yet there are also crossovers. Just think of European men's infatuation with young Polynesian women at the beginning of the 20th century, as we see in Paul Gauguin. I am not sure if those women found old white men equally beautiful. They had other things to offer, though.

GÜNTER M. ZIEGLER Christoph has an immediate response.

CHRISTOPH MARKSCHIES A very short remark, when looking at the tympanum of the portico of Frankfurt's Old Opera House, "Dem Wahren Schoenen Guten",[26] and looking to the Greek ideal of good and beautiful, you get the idea that these moralizing attempts or these attempts to equalize are – not always but often – ideas of the professionals, of the religious authorities, of the philosophical, political authorities. And the question is whether, in late modernity, these attempts to moralize broke down to a certain extent with the authorities.

---

26 Gerhard Kurz, Das Wahre, Schöne, Gute. Aufstieg, Fall und Fortbestehen einer Trias, Paderborn 2015.

OLAF DÖSSEL I have a very nasty picture of beauty, but it is related to the things that have been discussed right now. Let's say beauty is just a specific concert of neurons firing and chemicals being released in our brain. And this happens in a quite similar way in all our brains. So we can discuss it because we have the same kind of concert in our brains.
This concert originates partly from evolution, because evolution has made us think in a way, for example, that some human faces are beautiful, and this is enhanced during evolution, obviously. In addition, in a second part, it comes from what our brain has seen in our life because neurons are trained in some way. For example, after seeing pictures of faces a hundred times in a journal that are supposed to be beautiful, we also say it's beautiful. Yet, it's nothing less than a specific concert of neurons and chemicals.
That does not mean that I am not impressed personally by beauty, because I also have just such a brain. I am aware of that, I still enjoy this concert of neurons called beauty. But if you boil it down to natural sciences, it's just physics, electronics, and chemistry.

GÜNTER M. ZIEGLER I wanted to respond, but I also see Julia Fischer. You go first.

JULIA FISCHER I would say that this doesn't explain much. It's just a different level of description, but it doesn't explain why we have preferences. That would be in the realm of evolution, but simply saying it's just physics, for me, that would not be an explanation. It would be a different level of description.

OLAF DÖSSEL You just mentioned that evolution is convincing for you. But that was my argument.

GÜNTER M. ZIEGLER My understanding is that, if we come from mathematics and if David Harel says beauty is the first test, the understanding is that this concept of ideas and their beauty in mathematics is meant to be universal. I am not sure this is 100 percent clear or true, but there is a universal concept of beauty in mathematics and probably in all the other areas that we have looked at in the other contributions.
Beauty is always connected to the culture or to the historical development. And that means, for example, that this Central European idea of beauty is one

that's not shared in other cultures and other parts of the world and is far from universal. And I think it's part of our learning that exactly that is true.

HORST BREDEKAMP Some fifteen years ago there was – some of you were there – a three-day conference on symmetry at the Leopoldina. I think I was the last one to talk, and I thought I would bring in kind of adventure for the natural scientists that symmetry breaking is what makes art valuable. It's not symmetry. But over the three days, everyone was talking about this, symmetry and symmetry breaking.

In the human face, symmetry, absolute symmetry is a terrible disease and that's why a deviation is absolutely necessary for movement and for what you talked about, the heart. Absolute symmetry is like nothing. It is a disease.

And that is what Darwin took from Holgarth. Winfried Menninghaus brought that forward twenty years ago in this hall, showing that Darwin took all the discussions from the theory of art into the sexual selection principle.

And that's why he then founded – was able to found – the Max Planck Institute for Empirical Aesthetics. It was founded here in this hall because he showed that Darwin's concept of sexual selection is a summing up of 300 years of discussion of aesthetic beauty, which has to do with deviation.

Without deviation, no evolution. That is the point. The handicap principle explains many things, and Darwin talks about this. There are choices that go against strength, so against what is obviously the most powerful. And that explains why the most bizarre things exist through evolution. And that is what is the most surprising thing. What you say is true, but it doesn't explain this kind of absurdity, productive absurdity.

MARTIN QUACK I think everybody understood in David Harel's lecture why mathematics is beautiful. But I can actually add a non-beautiful solution to his problem of coinciding birthdays. When he bet 750 Euros, I could have actually immediately bet any amount, billions of Euros, that there are two people in this room that have the same birthday, because I happen to know by experiment that two people who are present have the same birthday.

So, there is a non-beautiful solution, which is experimental (although this may violate a rule of David Harel's game). And that brings me to science. Sometimes, it is thought that theoretical physics solutions must be beautiful mathematically. But there is actually a comment by Einstein that said essentially, "Well, beauty and elegance in theoretical physics, that's just for the tailors and hairdressers."

So, what he wanted to say, of course, is that what counts in natural science is whether experiment confirms the theory or not. Actually, on the symmetry that was mentioned by Horst Bredekamp, this meeting on symmetry at the Leopoldina is a classic example. In physics, things that were assumed to be naturally true, such as space-inversion symmetry, which Einstein himself pointed out must be naturally true, was later proven by experiment not to be available in nature, actually following his thinking in a sense.

So, whether in the end beauty (or *kalos* for beautiful) is relevant for the truth is not sure – some of you said *agathos*, but I think you wanted to say *alithinos*, right (for true, or *alaethaes, alaethinos* in ancient Greek)? Whether beauty and truth fit together is not certain. I think that something very ugly could be true, obviously.

GÜNTER M. ZIEGLER I think it is always difficult to prove things by quotes from Einstein because Einstein wrote so much, and you can always find a quote for the opposite as well. And in Jerusalem, they are just building a new museum for Einstein's legacy, and they will definitely have a beautiful building there.

But if you look at Einstein's equations for general relativity, mathematicians would say that they're absolutely beautiful. But the philosophical problem in the end is the question of why there is an equation in the first place and a beautiful and short and compact equation in the second place for something that in the end really describes space and gravitation?

And I think that, in the end, it is on the one hand not explained why math is so powerful there and why this can be cast in formulas. And on the other hand, it's not at all clear that, if you look at the connection between quantum mechanics and gravitation in the end, how should we know that there can be an equation or a set of equations or a compact mathematical theory that will describe it?

There is a book around called *Lost in Math*[27] that somehow claims that physicists got lost in trying to explain everything by mathematical equations in the

[27] Sabine Hossenfelder, Lost in Math: How Beauty Leads Physics Astray, Basic Books, New York 2018.

end. I only just got that yesterday, and I haven't read it yet. I don't believe that will be my thinking, but we'll see.

MARTIN QUACK There is actually an answer to why mathematics works. Of course, it's just a hypothesis. Human beings, following animals, plants, and early on bacteria have been trained for billions of years to confirm nature, right? And so our brain has basically been slowly trained over one million of years or ten millions of years to think in such a way that it works in nature.

GÜNTER M. ZIEGLER But works is one thing, and beautiful is the other thing, and there's a quote by Dirac which says it's important that the equations are beautiful. It's not so important that they fit the experiments.

MARTIN QUACK Well, Dirac – I know this is the opposition to Einstein, but Einstein definitely had the opposite philosophical view. Einstein and Dirac had opposite philosophical views about the importance of beauty. That's quite certain. Einstein always said that, in the end, it's experiment that has to decide, and that beauty doesn't decide.

GÜNTER M. ZIEGLER Beauty doesn't decide, I think that's just a beautiful way to end a beautiful session with thanks, a lot of thanks to David Harel and the other discussants.
And we won't cut off the discussion, but we will put it into the coffee break.

CHRISTOPH MARKSCHIES Yes, the coffee break is next, but I just need one short moment for some announcements beforehand. There are two things to be said at the end. The first thing is to thank Günter for bringing together the podium and for chairing the discussion in such impressive way. Many, many thanks, Günter!
And, David, I would like to emphasize that it was a moving sign of friendship between you and us, between the Israel Academy and our Academy, that you are the first speaker to come from abroad and speak English in this room during an internal plenary meeting of the Academy.
And I hope that we can continue these close relations through an exchange between many of us who are related to the Israel Academy, by our members

who are at the same time members of the Israel Academy, and by your members and so *todah rabah*, this was a beautiful lecture on beauty and a beautiful discussion, too – and a moving sign.
Next year, we will celebrate the 30th jubilee of our more than 320-year-old Academy. That's one of these riddles to solve. And there will be a discussion moderated by Julia and Ulrike concerning the future of the Academy – which will surely also be a beautiful discussion about beauty.

The Authors

BREDEKAMP, Horst | Professor of Art History, Humboldt-University Berlin; Senior Speaker, Cluster of Excellence Matters of Activity

FISCHER, Julia | Professor for Primate Cognition, University of Göttingen; Head of the Cognitive Ethology Laboratory, German Primate Center; Vice-President, Berlin-Brandenburg Academy of Sciences and Humanities

FREVERT, Ute | Director, Center for the History of Emotions, Max Planck Institute for Human Development; President, Max Weber Foundation – German Humanities Institutes Abroad

HAREL, David | The William Sussman Professorial Chair, Deptartment of Computer Science and Applied Mathematics, The Weizmann Institute of Science; President, Israel Academy of Sciences and Humanities

MARKSCHIES, Christoph | Professor of Ancient Christianity, Humboldt-University Berlin; President, Berlin-Brandenburg Academy of Sciences and Humanities; President, Union of the German Academies of Sciences and Humanities

SCHLAICH, Mike | Partner at schlaich bergermann partner; Professor and Chair of Conceptual and Structural Design, Technical University of Berlin

ZIEGLER, Günter M. | Professor of Mathematics; President, Freie Universität Berlin

berlin-brandenburgische
AKADEMIE DER WISSENSCHAFTEN
Jägerstraße 22/23 · 10117 Berlin · www.bbaw.de

ISBN: 978-3-949455-26-1