Exercise 1 (Sep 16)

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Exercise 1 (Girth and Short Cycles). The number of $2k$-cycles in a graph grows with number of edges. In this exercise, we will understand this function for the case $k = 2$.
(2a) Show that every graph with no 4-cycles has $O(n^{3/2})$ edges. Hint: A cherry in a graph is an ordered set $\langle u, \{v, w\}\rangle$ where v, w are neighbors of u. Bound the number of distinct cherries in the graph from below and above and use it to bound the number of edges in 4-cycle free graph.
(2b*) Prove that any n-vertex graph G with average degree Δ has $\Omega(\Delta^4)$ 4-cycles. *Remark:* The two claims above imply that the constant factor hidden in the $O(n^{3/2})$ edges is important. A graph with less than $c_1 \cdot n^{3/2}$ edges has *no* 4-cycle and every graph with at least $c_2 \cdot n^{3/2})$ edges has $\Omega(n^2)$ 4-cycles for $c_2 > c_1$.

Exercise 2 (Multiplicative Spanners). Show that the greedy spanner algorithm we saw in class has a runtime of $O(m \cdot n^{1+1/k})$. For simplicity, you may assume G being unweighted.