Distributed Graph Algorithms **Spring 2023**

Exercise 1: April 19

Lecturer: David Peleg

Exercise 1. Assume that every vertex knows the structure of the entire graph, and the communication model is CONGEST. Prove or disprove the following claims concerning a network $G(V, E)$.

(a) If there are at least k edge-disjoint paths of length at most d between the nodes v and w, then it is possible to send m messages from v to w in time $O(d + m/k)$.

(b) If $dist(v, w) = k$ and there are k^2 edge-disjoint paths between the nodes v and w, then it is possible to send k^2 messages from v to w in time $O(k)$.

Exercise 2. Consider the Multiple Messages (MM) problem with messages of size $O(\log n)$ on n-vertex networks $G(V, E)$ of diameter D, under the assumptions specified in class (namely, the availability of a mechanism for routing each message along a shortest path).

(a) Prove that the message complexity of MM has a universal lower bound of
$Message(MM, G) = \Omega(n \cdot D \cdot \log n)$,
or give a counter example.

(b) Prove that in the synchronous setting, $Time(MM, G) = O(n)$. Here you may assume that when two messages M_i and M_j are queued to be sent over the same outgoing link, M_i will be sent before M_j if and only if $i < j$.
Suggested approach: Prove (say, by induction on t) that for every $t \geq 1$, at the end of round t of the execution, the message M_i is either at distance at least $t - i + 1$ from the source r_1 or has already reached its destination v_i.