Exercise 2: May 26

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Low-Stretch Tree Embedding

Exercise 1. We showed in class how to compute a low-stretch tree embedding. A dual problem considers the construction of a *single* tree (either a subgraph of G or not) that has a small *average* stretch over all edges (u, v) in G. Formally, given an unweighted graph $G = (V, E)$ and a tree T with $V(G) \subseteq V(T)$, define the average stretch of T by:

$$1/|E(G)| \cdot \sum_{(u,v)\in E} \mathrm{dist}_T(u, v) \ .$$

(a) For a given even integer n, let W_n be the wheel graph consisting of n vertex ring C_n together with chords joining antipodal points on the ring. Find a tree $T \subseteq W_n$ with average stretch at most 8/3. (b) Show that the 2-dimensional $\sqrt{n} \cdot \sqrt{n}$ grid has a spanning tree with average stretch $O(\log n)$.

Exercise 2. We showed a randomized construction of a tree T such that $V(G) \subseteq V(T)$ and $\mathrm{dist}_G(u, v) \leq \mathbb{E}(\mathrm{dist}_T(u, v)) \leq \alpha \cdot \mathrm{dist}_G(u, v)$ for every $u, v \in V(G)$. In particular, the vertices of $V(G)$ are the leaves of T. Show that one can find in polynomial time another tree $T' = (V, E')$ (i.e., with $V(T') = V(G)$) such that $\mathrm{dist}_T(u, v) \leq \mathrm{dist}_{T'}(u, v) \leq 4\mathrm{dist}_T(u, v)$.

Routing Schemes

Exercise 3. Describe an efficient routing scheme for the unweighted $\sqrt{n} \times \sqrt{n}$ 2-dimensional grid. The labels and the routing tables should be of size $O(\log n)$ bits. Bonus: extend it to the d-dimensional n-vertex hypercube for $n = 2^d$.