Exercise 2: April 24

Lecturer: Merav Parter

Distance Oracles

We presented in class a $(2k-1)$ approximate distance oracle scheme by Thorup and Zwick [TZ05]. In this exercise, we will provide a tighter analysis for this scheme in certain settings.

Exercise 1. We analyzed the query algorithm and showed that given an arbitrary query pair u, v, the output distance estimate $\widehat{\delta}(u,v)$ can be bounded by $\widehat{\delta}(u,v) \leq (2k-1) \cdot \texttt{dist}(u,v,G)$. We are now interested in whether a tighter analysis of the stretch can be obtained for certian query pairs. Suppose that the query pair u, v is such that $u \in A_i$ for $i \geq 1$. Prove or Refute: for such a query, the query algorithm returns a distance approximation which is strictly better than $2k-1$. In particular, bound the stretch factor $f(k,i)$ such that $\widehat{\delta}(u,v) \leq f(k,i) \cdot \texttt{dist}(u,v,G)$, for $u \in A_i$.

Exercise 2. The distance oracle presented in class supported distance queries u, v for any $u, v \in V \times V$. Our goal is to construct a *smaller* oracle of size $o(n^{1+1/k})$ that would handle only a subset of query pairs. In particular, you are now given a subset of vertices, called hereafter, *sources*, $S \subseteq V$ and it is required to design a *source-wise* approximate distance oracle scheme. In such a scheme, the query algorithm receives only queries of the form $u, v \in S \times V$ (i.e., your oracle should only answer distance queries between one of the sources and some other vertex in the graph). Show that in such a case, the preprocessing algorithm can be adapted to yield a data structure of size $\widetilde{O}(k|S|^{1/k} \cdot n)$. Your answer should include a modified preprocessing algorithm, query algorithm and a correctness analysis. Hint: Modify the definition of A_0 and the sampling probabilities.

References

[TZ05] Mikkel Thorup and Uri Zwick. Approximate distance oracles. *Journal of the ACM (JACM)*, 52(1):1–24, 2005.