Exercise 3: June 27

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Cut Sparsification

Exercise 1. You are given a graph G that has a good *edge-expansion* such that for every $S \subset V$, $|S| \leq n/2$, it holds that:

$$|E(S, V \setminus S)|/|S| \geq \alpha, \quad \text{where} \quad \alpha = \Omega(\log n) \ .$$

Show that if we sample each edge $e \in G$ with probability $p = \Omega(\log n/(\alpha \cdot \epsilon^2))$ then all cuts are preserved within $(1 \pm \epsilon)$ of their expectation with high probability (at least $1 - 1/n^5$). That is, show that w.h.p. for every $S \subseteq V$, $|S| \leq n/2$, the number of sampled edges in the cut $(S, V \setminus S)$ is $(1 \pm \epsilon) \cdot p \cdot |E(S, V \setminus S)|$. Instructions: you should *not* use the cut counting argument that we saw in class, i.e., do not use the fact that there are at most $n^{O(\alpha)}$ cuts of size $\alpha \cdot c$ where c in the min-cut in G.

Reachability Shortcuts

Exercise 2 (Shortcuts for Paths). We presented in the talk a simple algorithm for computing 2-shortcuts for n-length dipaths with $O(n \log n)$ edges. Show that for every given diameter bound d and an n-length dipath P one can compute a d-shortcut for P with $\widetilde{O}(n/d)$ edges.

Exercise 3 (Subsetwise Shortcuts). Let $TC(G)$ denote the transitive closure of a graph G. Given a graph $G = (V, E)$, a subset $S \subseteq V$ and an integer d, a set of edges $H \subseteq TC(G)$ is an (S, d)-shortcut, if for every $u, v \in V$, there is a u-v path $P_{u,v}$ in $G \cup H$ that contains at most d vertices from S. Show that given an n-vertex m-edge DAG G, a subset $S \subseteq V$ and a diameter bound d, one can compute an (S, d)-shortcut H with $\widetilde{O}(|S| + |S|^2/D^3)$ edges.