# Abstracts SPD8 - May 20, 2021

Prof. Misha Sodin, Tel Aviv University: *Power series with random and pseudo-random coefficients*

Though the study of the influence of the arguments of the coefficients of power series on their zero distribution was initiated long time ago by Levy, Littlewood, and Offord, this area remains a terra incognita to this day.

Recently we revealed an intimate relation between the zero distribution and the pair correlations (the Wiener spectrum) of the coefficients, which allows us to treat examples of a very different nature (stationary ergodic sequences, Besicovitch almost-periodic sequences, and sequences of arithmetic origin), both on a macroscopic and on a local scale.

The talk will be based on joint works with Jacques Benatar, Alexander Borichev, and Alon Nishry.

Andrea Sartori, Tel Aviv University: *Expected nodal volume for random non-gaussian band-limited functions*

A central object in the study of Laplace eigenfunctions is their zero set, sometimes referred to as nodal set. I this talk I will discuss the nodal volume (volume of the nodal set) of random linear combinations of Laplace eigenfunctions on a compact manifold, also known as band-limited functions. In particular, I will show that the expectation of the said nodal volume is independent of the distribution of the coefficients, provided they have a finite second moment. The talk is based on recent work with Z. Kabluchko and I. Wigman.

Roy Shmueli, Tel Aviv University: *The expected number of roots of random polynomials over the field of p-adic numbers*

There is a classical problem of estimating the expected number of real roots of a random polynomial, going back at least to Littlewood-Offord. In this talk we shall present a p-adic version of this problem and a result estimating of the expected number of roots for a general family of distributions, and several corollaries of the theorem for specific distributions such as a random Littlewood polynomial (polynomial whose coefficients are +1 or -1).

Oren Yakir, Tel Aviv University: *Random polynomials near the unit circle*

It is well known that a random polynomial with iid coefficients has most of its roots close to the unit circle. Recently, Michelen and Sahasrabudhe found the limiting distribution for the closest root to the unit circle, in the case of Gaussian coefficients. We give a different proof of their result, which shows that the limit distribution is in fact universal (i.e. remains true for general coefficient distribution). Our new proof is inspired by earlier works of Konyagin and Schlag on the minimum modulus of the polynomial on the unit circle itself.

Joint works with Nick Cook, Hoi Nguyen and Ofer Zeitouni.

Michael Chapman, Hebrew University: *New cutoff results*

Total variation cutoff of simple random walks on graphs received a lot of attention in recent years. In this talk I am going to briefly introduce the subject, review known results in the field and discuss some new ones. This talk is based on joint works with Ori Parzanchevski and Yuval Peled.

Pengfei Tang, Tel Aviv University: *Two conjectures about return probability on transient, transitive graphs*

Consider simple random walk $(S_n)$ on a transitive graph with spectral radius $r$. Let $u_n = P[S_n = S_0]$ be the n-step return probability. It is a folklore conjecture that on transient, transitive graphs $u_n/r^n$ is at most of the order $n^(-3/2)$. We prove this conjecture for graphs with a closed, transitive, amenable and nonunimodular subgroup of automorphisms. We also study the first return probability $f_n$. We conjecture that for transient, transitive graphs, $f_n$ is the same order of $u_n$. For a graph $G$ with a closed, transitive, nonunimodular subgroup of automorphisms, we show that there is a positive constant $c$ such that $f_n \geq u_n/cn^c$.

Shaked Leibzirer, Technion: *On the eigenvalues in the Linial–Meshulam model*

Denote by $A$ the adjacency operator of $X(d,p,n)$, the Linial-Meshulam model for random $d$-dimensional simplicial complexes on $n$ vertices, where each $d$-cell is added independently with probability $p \in [0,1]$ to the complete $(d-1)$-skeleton. We consider the matrix $H$, a sparse random matrix, which generalizes $\mathcal{A} := 1/\sqrt{np(1-p)}(A - E[A])$, the centered and normalized adjacency matrix of $A$. While the non-zero entries of $A$ are normalized Bernoulli $p$ random variables, those of $H$ are any bounded random variables with the same law. We show that for any positive constant $C$, all large enough $n$ (depending on $d$ and $C$) and any integer $k := k(n)$ satisfies $Clog(n) \leq k$, we have that $E([\|H\|_{S_{2k}}^{2k}])^{1/2k} \leq \Phi(\theta_k, \theta_k^*)$, where $\|\cdot\|_{S_{2k}}$ denotes the $2k$-Schatten norm, $\theta_k, \theta_k^*$ are explicit functions of the entries of $H$, and $\Phi(\cdot,\cdot)$ is an explicit function of $\theta_k$ and $\theta_k^*$. We use this bound in order to prove that under the assumption $n\mathrm{Var}(Z) \gg \log(n)$, one has $lim_{n\to\infty} E[\|H\|_2] = 2\sqrt{d}$, and and $lim_{n\to\infty}\|H\|_2 = 2\sqrt{d}$, P-almost surely, where $Z$ is a random variable with the same law as the entries of $H$. The main tool of the proof is a generalization of [1], Theorem 4.8, which is based on combinatorial arguments and the exploitation of the dependent structure of the entries of $H$, which is governed by the simplicial structure.

[1] Rafal Latala, Ramon Van Handel, and Pierre Youssef. The dimension-free structure of nonhomogeneous random matrices. Invent. Math., 214(3):1031-1080, 2018.

Sarai Hernandez-Torres, Technion: *The time constant of finitary interlacements*

The finitary random interlacement $FRI(u, T)$ is a Poisson point process of geometrically killed random walks on $Z^d$, with $d \geq 3$. The parameter $u$ modulates the intensity of the point process, while $T$ is the expected path length. Although the process lacks global monotonicity on $T$, $FRI(u, T)$ exhibits a phase transition. For $T > T^*(u)$, $FRI(u, T)$ defines a unique infinite connected subgraph of $Z^d$ with a chemical distance. We focus on the asymptotic behavior of this chemical distance and-in particular-the time constant function. This function is a normalized limit of the chemical distance between the origin and a sequence of vertices growing in a fixed direction. In this sense, the time constant function defines an asymptotic norm. Our main result is on its continuity (as a function of the parameters of FRI). This talk reports on work in collaboration with Eviatar Procaccia and Ron Rosenthal.

Daniel Hadas, Tel Aviv University: *Random high-density packings of 2x2 tiles on the square lattice*

Consider random configurations of disjoint $2x2$ tiles positioned in integer coordinates in a bounded domain in $R^2$, where the probability for the appearance of a configuration is proportional to $\lambda^n$, with $n$ being the number of tiles in the configuration. This is called the $2x2$-Hard-Squares model with fugacity $\lambda$. Many similar hard-core lattice gases undergo a phase transition: at low fugacities the random configuration is disordered with exponential decay of correlations while at high fugacities the random configuration globally approximates a single optimally-packed configuration. This paradigm is inapplicable to the $2x2$-hard-squares model due to a "sliding" degree of freedom in the optimal packing, and the question of its high-fugacity behavior remained open. We show that at high fugacities, the $2x2$ hard-squares model exhibits a columnar phase: the tiles either preferentially occupy the even rows, the odd rows, the even columns or the odd columns.

Paul Dario, Tel Aviv University: *Localization and delocalization for random-field random surfaces*

In their seminal work, Imry and Ma predicted that the addition of a quenched random external field causes the disparition of first-order phase transitions in low-dimensional spin systems. These predictions were confirmed by Aizenman and Wehr for a large class of spin systems and recently quantified in the case of the random-field Ising model. While the Imry-Ma phenomenon has mostly been investigated in the context of compact spin spaces, it has been recognized that a related effect occurs for effective interface models of $\nabla\phi$ type. In this talk, we will study how the addition of a quenched random external field modifies the qualitative properties of these random interface models and present some related open problems.

Chanwoo Oh, Technion: *Metastability of zero range processes via Poisson equations*

Certain zero range processes on a finite set exhibits metastability. Most of the time nearly all particles of the zero range process are at one single site, and the site of condensate asymptotically behaves as a Markov chain. In this talk, we prove the metastability of zero range processes on a finite set with an approach using the Poisson equation. This approach doesn't need precise estimates of capacities and can be applied for both reversible and non-reversible cases. This talk is based on the joint work with F. Rezakhanlou.

Keren Mor, Tel Aviv University: *On the zeros of hyperbolic Gaussian Analytic functions*

We consider the family $F_L$ of Gaussian analytic functions in the unit disk, distinguished by the invariance of their zero set with respect to the hyperbolic isometries. Peres and Virág showed that for $L = 1$ (and only then) the zero set forms a determinantal point process, making many explicit computations possible. I will talk about the asymptotic probability of the rare event where there is an overcrowding of the zeros, when $L > 0$ is arbitrary. Curiously, contrary to the much better understood planar model, it appears that for $L < 1$ the probability of overcrowding is much less than the hole probability.

Shay Sadovsky, Tel Aviv University: *Transportation with obstacles: Optimal transport for non-traditional costs*

The classical problem of optimal transport deals with finding a map from one distribution to the other, which is minimal with respect to some finite-valued cost function. We will give a short review of this problem, and the related notions of c-transform and c-subgradients. We will see that the solution to the classical problem does not apply to the non-traditional case when the cost function is allowed to attain the value infinity - i.e. when some points may not be mapped to others. We will give conditions under which there exists a solution to the non-traditional problem, and prove that it is a c-subgradient of some function. Based on joint work with Shiri Artstein-Avidan and Kasia Wyczesany.

Eyal Castiel, Technion: *Fluid limits of Queue-based CSMA, homogenization and reflection* In this talk, we will discuss the fluid limits of a queueing process with the QB-CSMA scheduling policy. Introduced in 2009, this algorithm aims at mimicking the behavior of the celebrated Max-Weight algorithm by Tassiulas and Ephremides in a fully distributed fashion. The key element of the analysis is a fully coupled stochastic averaging principle where the schedule evolves much faster than queue lengths and we can replace the service rates by an invariant measure 'adapted' to the current queue lengths. This approximation fails in a neighborhood of zero but we will be able to overcome this difficulty in the case of a complete interference graph through a coupling argument.

Inbar Seroussi, Weizmann: *Lower Bounds on the Generalization Error of Neural Network algorithms*

Deep learning algorithms operate in regimes that defy classical learning theory. Neural networks architectures often contain more parameters than training samples. Despite their huge complexity, the generalization error achieved on real data is small. In this talk, we aim to study generalization properties of algorithms in high dimension. Interestingly, we show that algorithms in high dimension require a small bias for good generalization. We show that this is indeed the case for deep neural networks in the overparametrized regime. In addition, we provide lower bounds on the generalization error in various settings for any algorithm. We calculate such bounds using random matrix theory (RMT). These bounds are particularly useful when the analytic evaluation of standard performance bounds is not possible due to the complexity and non-linearity of the model. The bounds can serve as a benchmark for testing performance and optimizing the design of actual learning algorithms. (Joint work with Ofer Zeitouni)

Zhenyao Sun, Technion: *Noise effect on the speed of stochastic reaction-diffusion equation with Holder drift*

We consider the $[0, 1]$-valued solution $(u_{t,x} : t \geq 0, x \in R)$ to the one-dimensional stochastic reaction-diffusion equation with Wright-Fisher noise: $\partial_t u = \partial_x^2 u + f(u) + \epsilon\sqrt{u(1-u)}\dot{W}$. Here, $W$ is a space-time white noise, $\epsilon > 0$ is the noise strength, and $f$ is a continuous function on $[0, 1]$ satisfying $|f(z)| \leq C\sqrt{z(1-z)}$. Note that $f$ is not necessarily Lipschitz. We assume the initial data satisfies $1 - u_{0,-x} = u_{0,x} = 0$ for $x$ large enough. Recently, it was proved in [1] that the front of $u_t$ propagates with a finite deterministic speed $V_{f,\epsilon}$, and under slightly stronger conditions on $f$, the asymptotics of the $V_{f,\epsilon}$ was derived as the noise strength $\epsilon$ approaches $\infty$. In this talk we report a result on the asymptotic behavior of $V_{f,\epsilon}$ as the noise strength $\epsilon$ approaches 0: For a given $p \in [1/2, 1)$, if $f(z)$ is non-negative, and is comparable with $z^p$ for sufficiently small $z$, then $V_{f,\epsilon}$ is comparable with $\epsilon^{-2\frac{1-p}{1+p}}$ for small $\epsilon$. This is based on my ongoing work with Clayton Barnes and Leonid Mytnik.

[1]: Mueller, C., Mytnik, L. and Ryzhik, L.: The speed of a random front for stochastic reaction-diffusion equations with strong noise. To appear in Commun. Math. Phys. (2021).

Ohad Klein, Bar Ilan University: *On the distribution of randomly signed sums*

We discuss random variables $X = suma_i x_i$, where $a_i$ are constants and $x_i$ are uniformly random i.i.d signs (in $\{-1, 1\}$). We survey several concentration properties of these variables, including the resolution of Tomaszewski's conjecture.

Peter Ralli, Weizmann Institute: *Graph powering and generalized Ramanujan graphs*

The r-th power of a graph modifies a graph by connecting every vertex pair within distance r. This talk gives a generalization of the Alon-Boppana Theorem for the r-th power of graphs, including irregular graphs. This leads to a generalized notion of Ramanujan graphs, those for which the powered graph has a spectral gap matching the derived Alon-Boppana bound. In particular, we show that certain graphs that are not good expanders due to local irregularities, such as Erdos-Renyi random graphs, become almost Ramanujan once powered. We also see that sparse Erdos-Renyi random graphs with an adversary modifying a subgraph of polylog(n) vertices are still almost Ramanujan. As an application, this gives robust community testing for different block models. Joint work with Emmanuel Abbe.

Lior Tenenbaum, Technion: *Asymptotic number of trees in random Steiner complexes*

A spanning tree $T$ of a graph $G$ is a sub-graph of $G$ with the same vertex set as $G$ that is a tree. In 1981, McKay proved an asymptotic result regarding the number of spanning trees in random $k$-regular graphs. In this talk we discuss an analogous result for random high-dimensional $k$-regular simplicial complexes, called random Steiner complexes, showing that the weighted number of simplicial spanning trees in such complexes converges asymptotically almost surely to an explicit constant $c_{d,k}$, when $n$ tends to infinity, provided $k \geq 2d^2 + 2d\sqrt{d^2-1}$. A key ingredient in the proof is the local convergence of such random complexes to the $d$-dimensional, $k$-regular arboreal complex, which allows us to generalize Mckay's result regarding the Kesten Mckay distribution.