

# Cognitive Sub-Nyquist Collocated MIMO Radar Prototype with Clutter Removal

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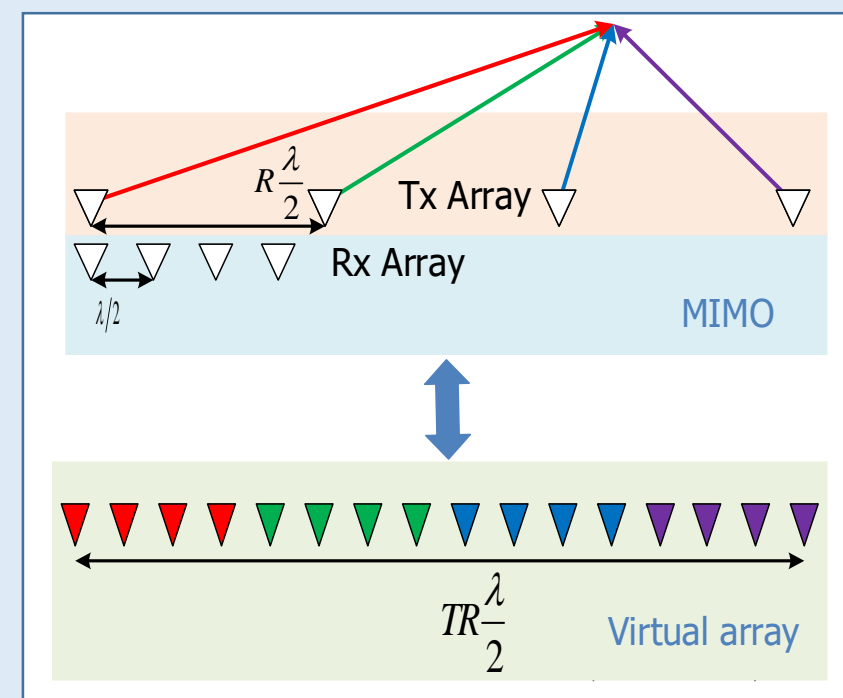


## Main Contributions

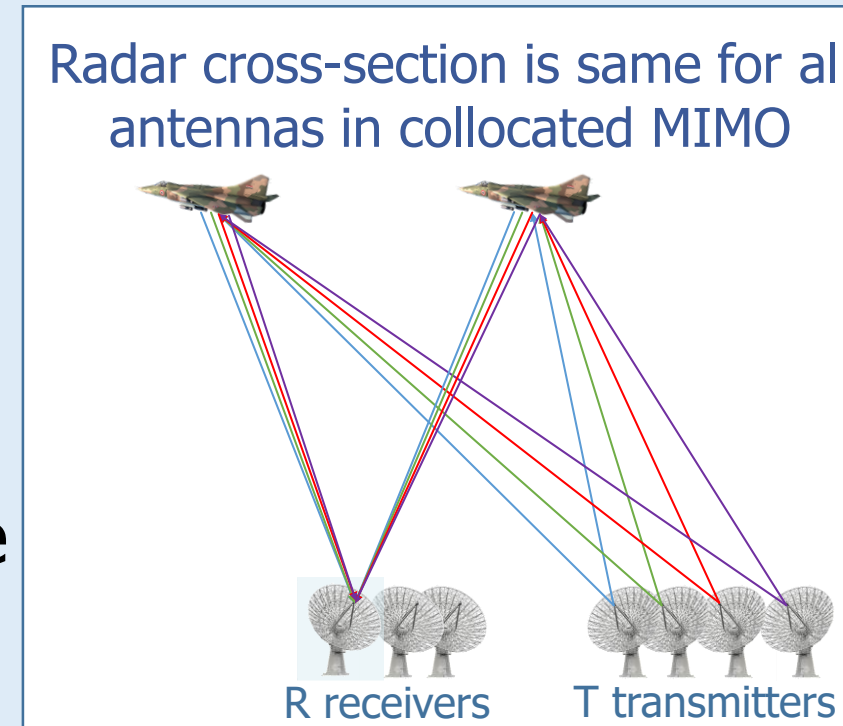
- Prototype realizes both spatial and temporal sub-Nyquist sampling in a MIMO radar without loss of angular and range resolution
- Sub-Nyquist 4x5 MIMO array shows same detection performance as Nyquist 8x10 ULA
- The Reduction rate is 75%
- Cognitive transmission is employed to further enhance SNR for sub-Nyquist arrays

## Conventional Collocated MIMO

MIMO array with fewer elements has same spatial resolution as a virtual array with more elements



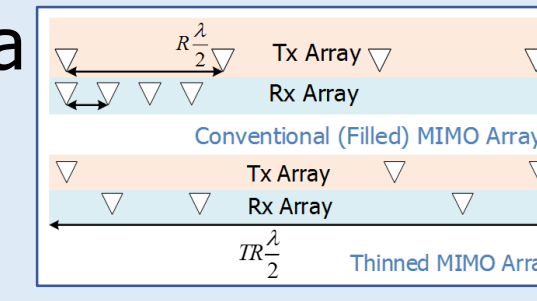
MIMO transmits orthogonal waveforms and processes linear combination of echoes received due to each waveform



## Sub-Nyquist MIMO

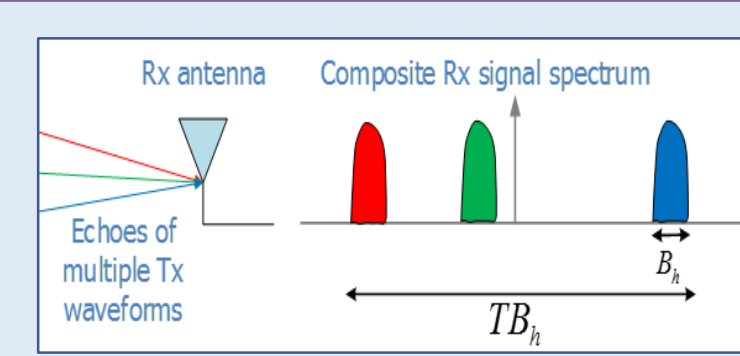
### Spatial Sub-Nyquist

- Less antenna elements via randomly thinned arrays
- Same angular resolution as of virtual array



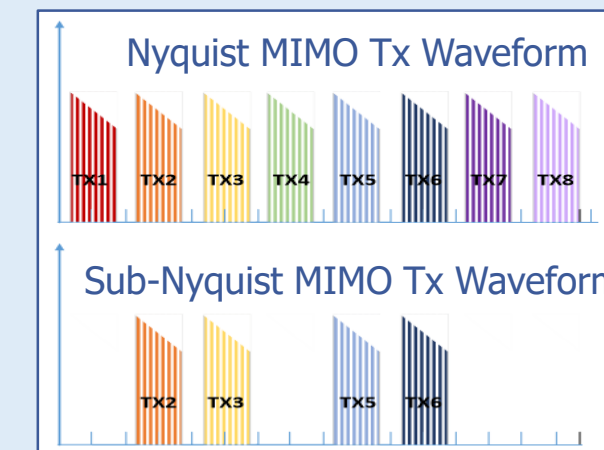
### Temporal Sub-Nyquist

- Reduced sampling rate at each Rx
- Same range resolution as that of Nyquist bandwidth  $TB_n$



### Cognitive Transmission

- Entire power is focused in only few narrow subbands
- High SNR at receiver



## Signal Model and Recovery

- Received signal for P pulses at the qth antenna after demodulation:

$$x_q(t) = \sum_{p=0}^{P-1} \sum_{m=0}^{M-1} \alpha_m h_m(t - \tau_l - p\tau) e^{j2\pi\beta_{m,q}\theta_l} e^{j2\pi f_m p\tau}$$

- Fourier coefficients of the mth transmitter channel at the qth receiver:

$$y_{m,q}^p[k] = \sum_{l=1}^L \alpha_l e^{j2\pi\beta_{m,q}\theta_l} e^{-j\frac{2\pi}{T}(k+fm\tau)\tau_l} e^{-j2\pi f_m \tau_l} e^{j2\pi f_m^p \tau}$$

- Doppler focusing for a specific frequency  $\nu$

$$\Phi_{m,q}^{\nu}[k] = \sum_{l=1}^L \alpha_l e^{j2\pi\beta_{m,q}\theta_l} e^{-j\frac{2\pi}{T}(k+fm\tau)\tau_l} \times \begin{cases} 1 & |f^p - \nu| < 1/2P\tau \\ 0 & \text{else} \end{cases}$$

- Use OMP for simultaneous sparse 3D recovery with focusing

## Technical Specification

	Nyquist (Mode 1)	Sub-Nyquist (Mode 3)	Reduction
BW per Tx (incl. guard-bands)	15 MHz	3 MHz	80%
BW per Tx (excl. guard-bands)	12 MHz	3 MHz	75%
Temporal sampling rate	30 MHz	7.5 MHz	75%
Spatial sampling	8x10	4x5	50%
# Tx/Rx channels	80	20	75%
Total Bandwidth	120 MHz	12 MHz	90%

## Clutter Model

The received signal  $r_q(t)$  at the qth antenna is  $r_q(t) = x_q(t) + y_q(t) + n_q(t)$

The clutter amplitude in the angular range  $[\theta - \epsilon, \theta + \epsilon]$  is

$$a_c = \frac{1}{2\pi} \int_{\theta - \epsilon}^{\theta + \epsilon} S_c(\theta) d\theta$$

where  $S_c(\theta)$  is the clutter amplitude density at angle  $\theta$ .  $a_c \sim N(a_c, \sigma_c^2)$

The delays  $\tau_c \sim U(0, \tau)$ , DoA  $\phi_c \sim U(-1, 1)$  and clutter Doppler spectrum  $v_n \sim N(v_n, \sigma_d^2)$  are i.i.d.

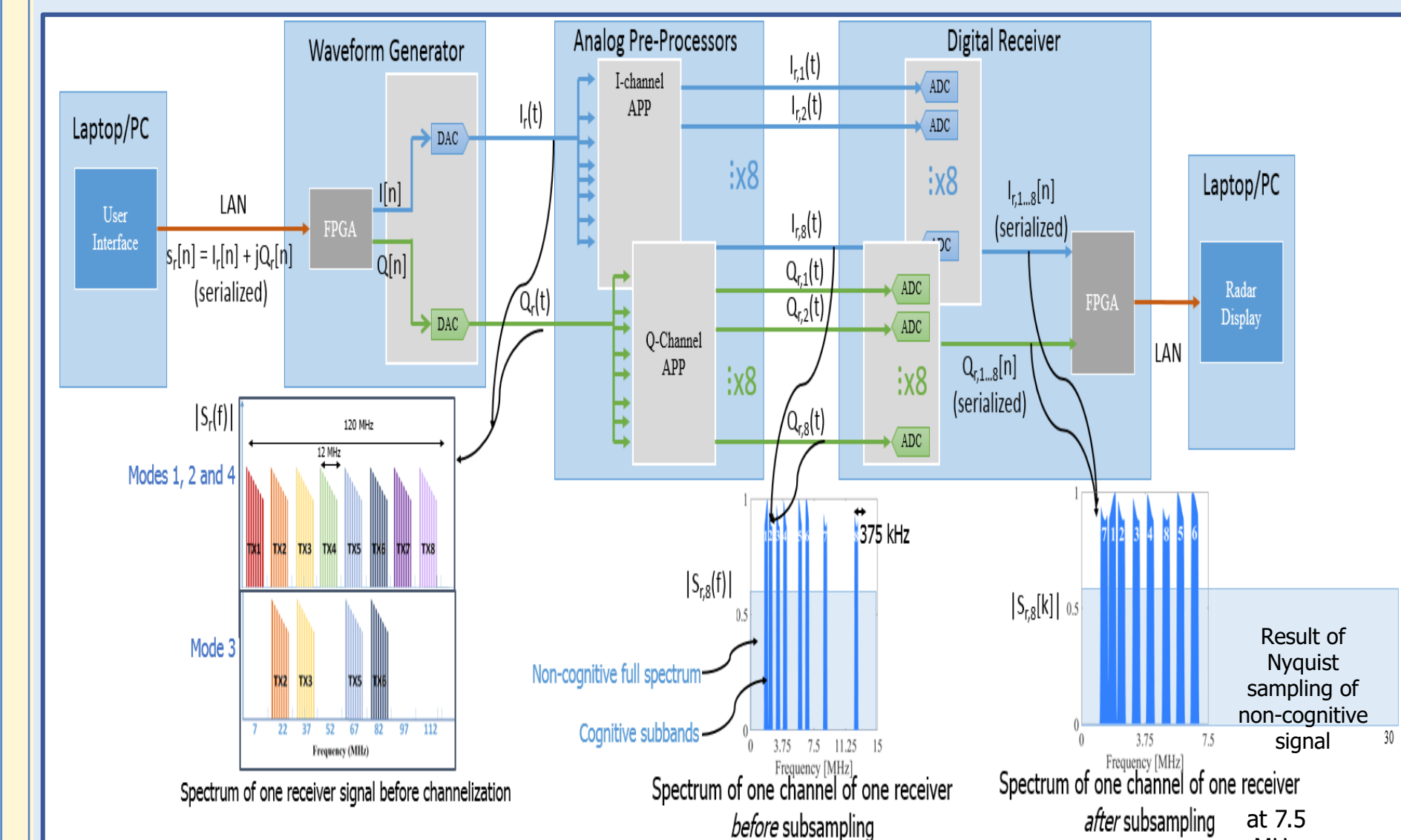
## Clutter Mitigation

$Z_m^T = P \times K \times Q$  is the clutter plus noise matrix with columns  $z_{m,1}, z_{m,2}, \dots, z_{m,KQ}$   
m is number of Transmitters  
P is number of pulses  
Q is number of receivers  
K is randomly chosen group of coefficients

### Four stages for clutter whitening

- Subtracting the mean:**  $R_m^T - E[Y_m^T + N_m^T] = X_m^T + Z_m^T - E[Y_m^T + N_m^T]$  or  $\tilde{R}_m = X_m^T + \tilde{Z}_m$
- Reshaping:**  $\tilde{Z}_m$  is reshaped from a  $P \times KQ$  matrix to  $PQ \times K$  matrix or  $Z_m$  is a  $PQ \times K$  clutter-plus-noise matrix.  $\tilde{R}_m$  is reshaped to  $\tilde{R}_m$
- Whitening:** The covariance matrix of the qth column  $\hat{z}_{pq,k}$  of  $\tilde{Z}_m$  is a  $PQ \times PQ$ . Toeplitz matrix:  $D_m = E[\hat{z}_{pq,k} \hat{z}_{pq,k}^T]$   
Whitened measurements:  $D_m^{-1/2} \tilde{R}_m = D_m^{-1/2} X_m^T + W_m^T$  where  $W_m$  is white noise.
- Signal recovery:** Transpose the above equation to get the desired form and proceed with the Doppler focusing followed by the sub-Nyquist MIMO recovery algorithm.

## Overview of Hardware Architecture



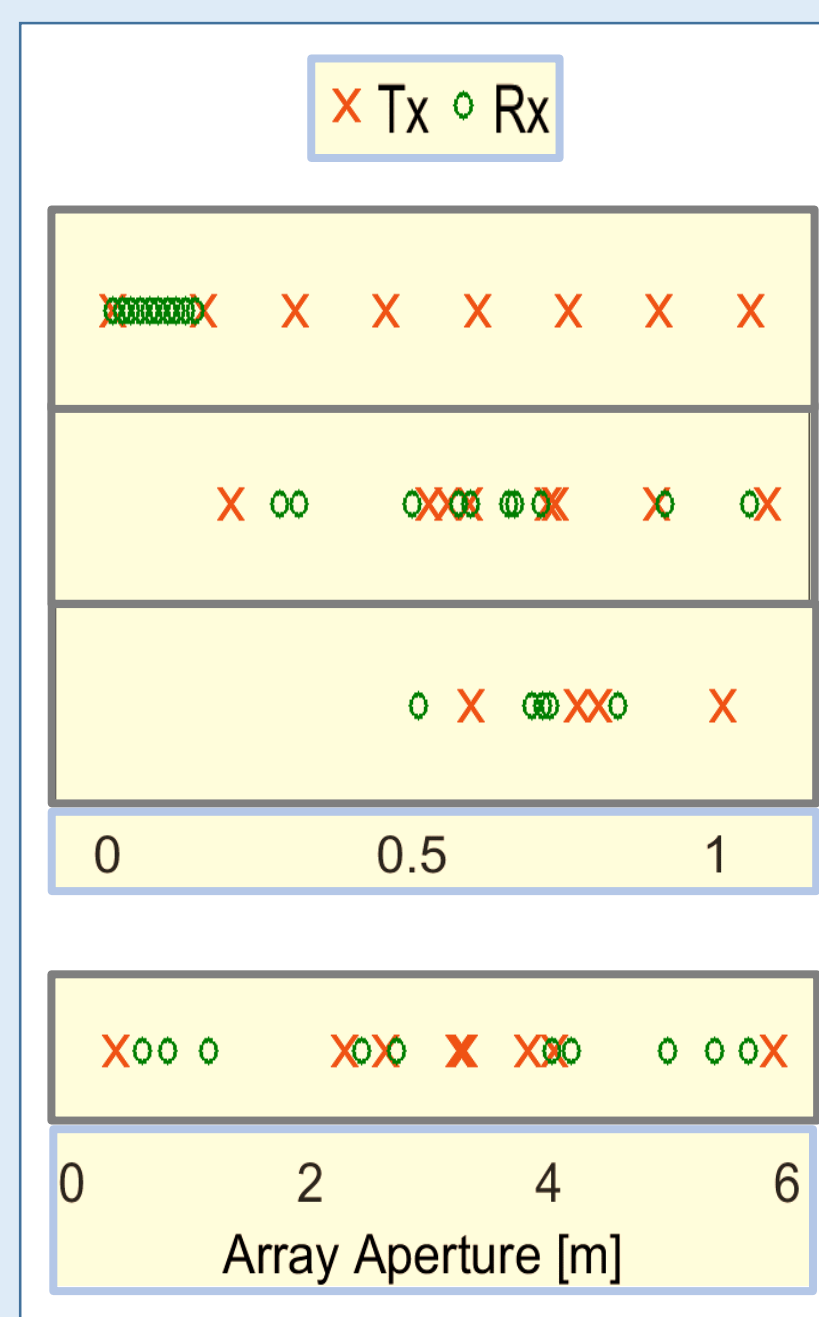
## Array Modes

Mode 1: 8x10 Filled uniform array

Mode 2: 8x10 Filled random array

Mode 3: 4x5 Thinned random array (~Virtual 8x10 ULA) Spatial sub-Nyquist

Mode 4: 8x10 Thinned random array (~Virtual 20x20 ULA)



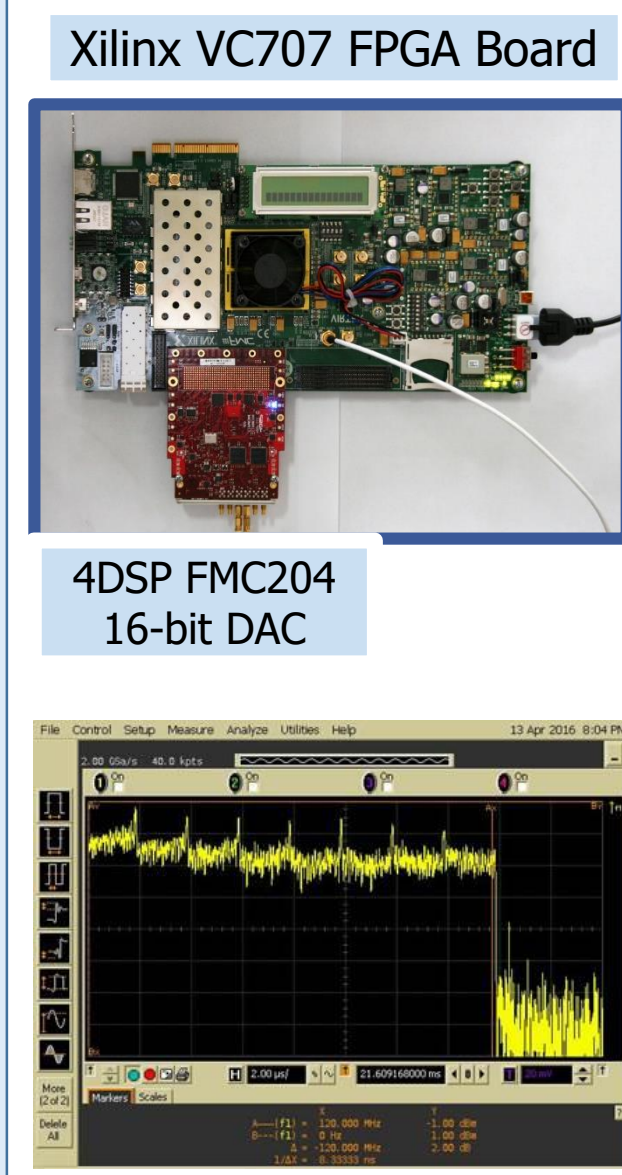
## Waveform Generator

- Total BW, 8 Tx: 120 MHz  
3 MHz guard-bands

- Eight 375 kHz cognitive slices per Tx

- Cognitive BW, 1 Tx: 3 MHz (= 8 x 375 kHz)

- BW reduction, 1 Tx (excl. guard-bands): 75% (3 of 12 Mhz)

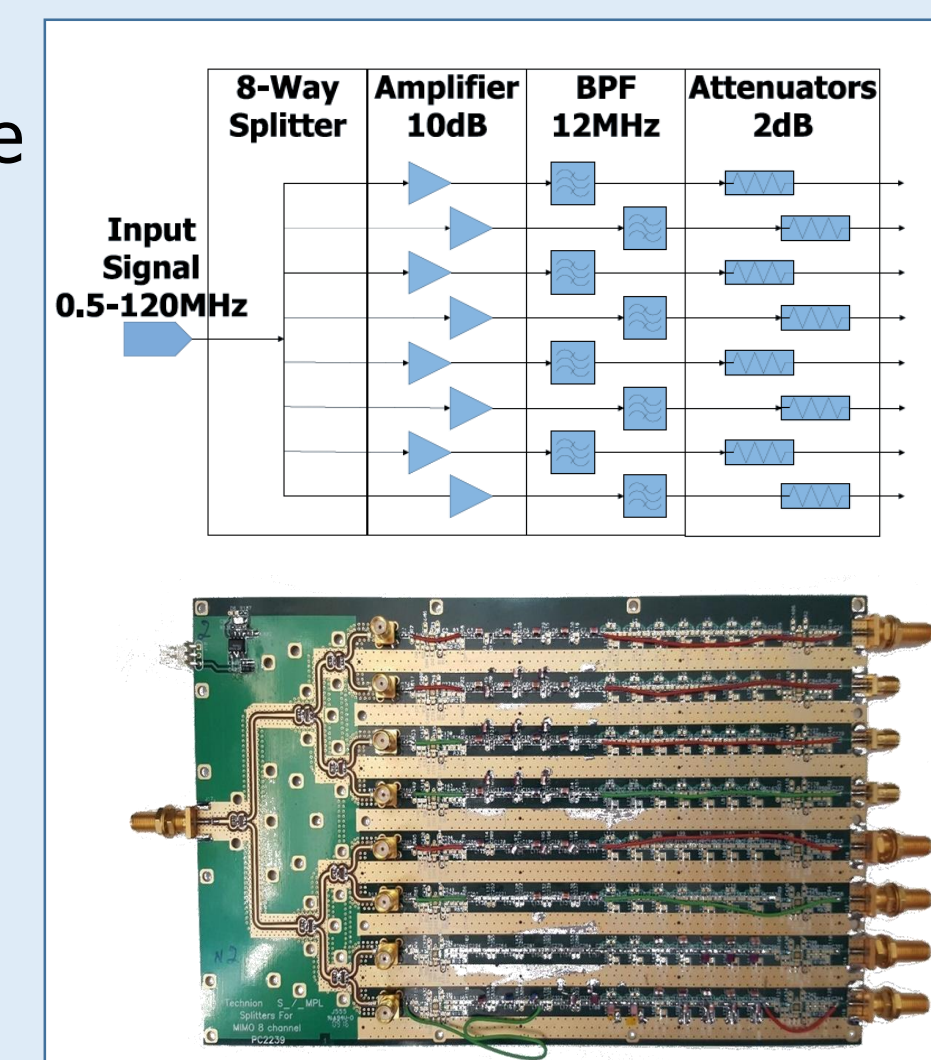


## Analog Pre-Processor (APP)

- APP filters the receiver data into eight channels

- APP card in a single chassis

- BPFs have ~30 dB stopband attenuation to mitigate subsampling noise

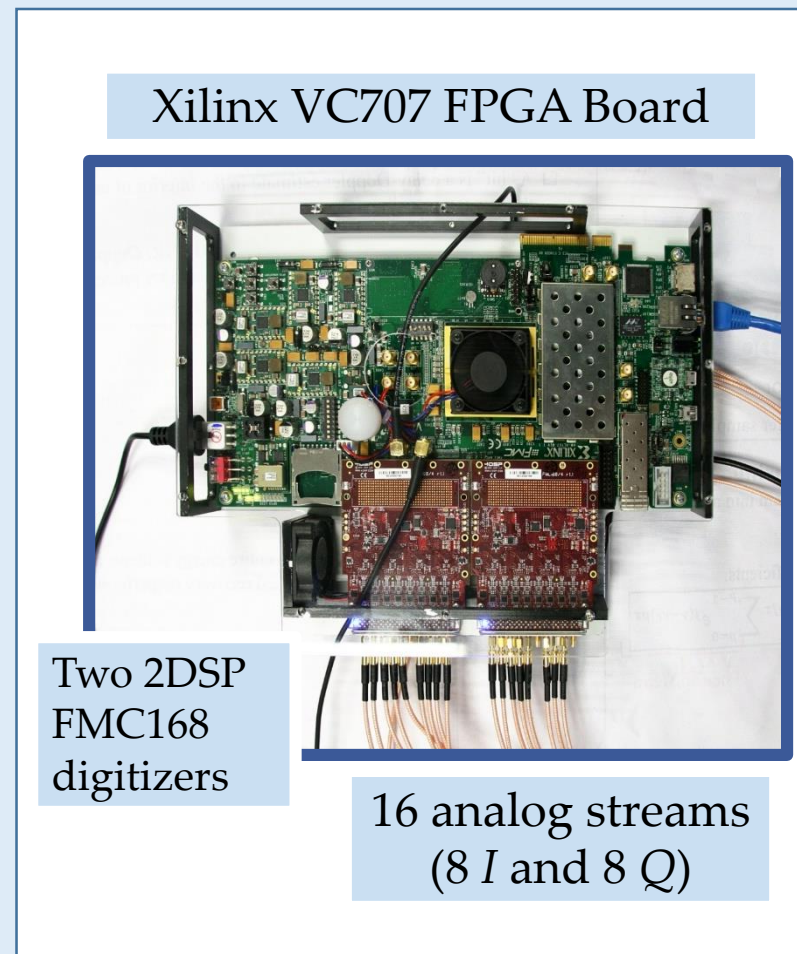


## Digital Receiver

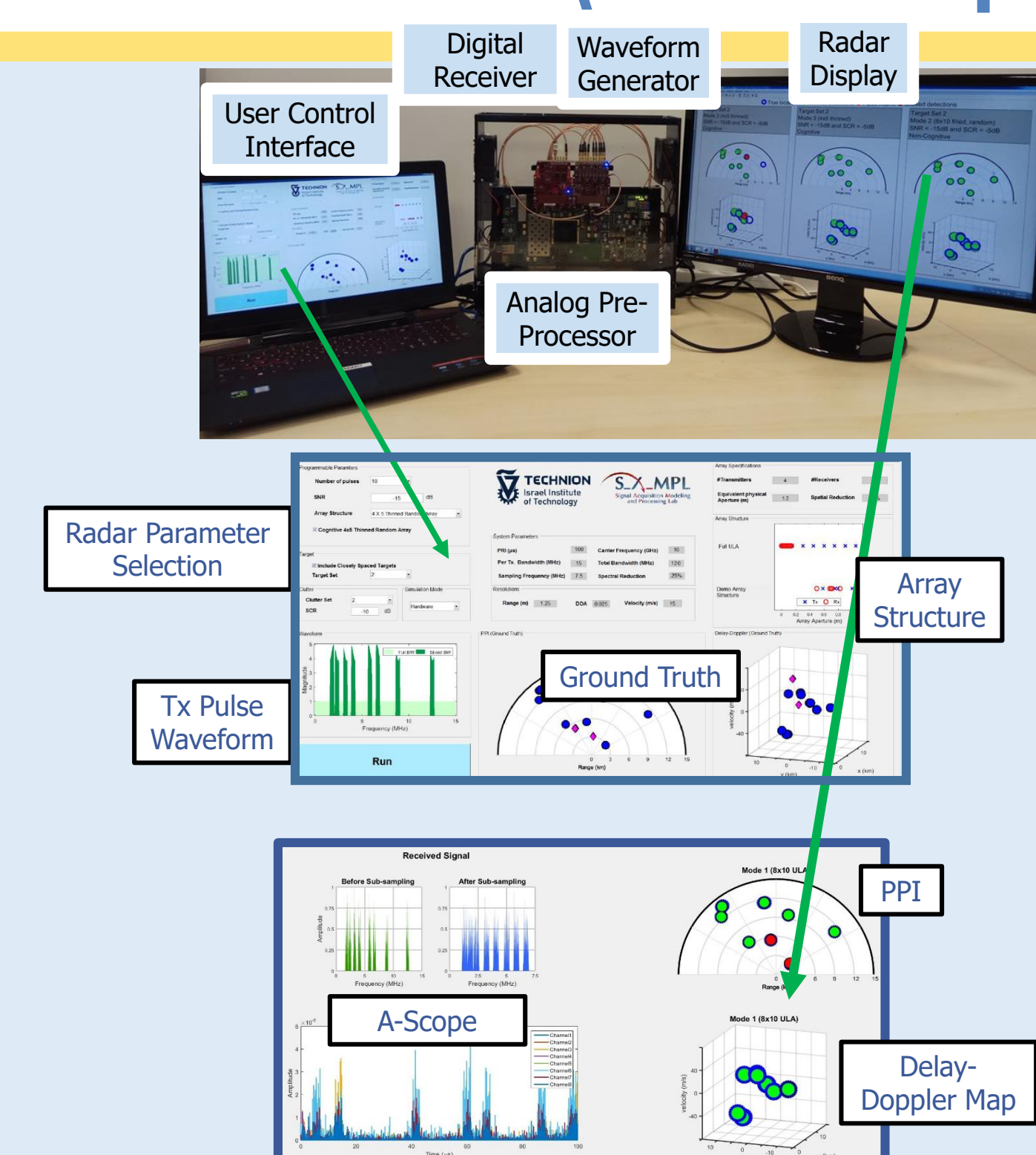
- Two 16-bit eight-channel digitizers for I and Q streams

- Sub-Nyquist sampling rate: 7.5 MHz/channel

- Signal BW with guard-bands: 30 MHz/channel

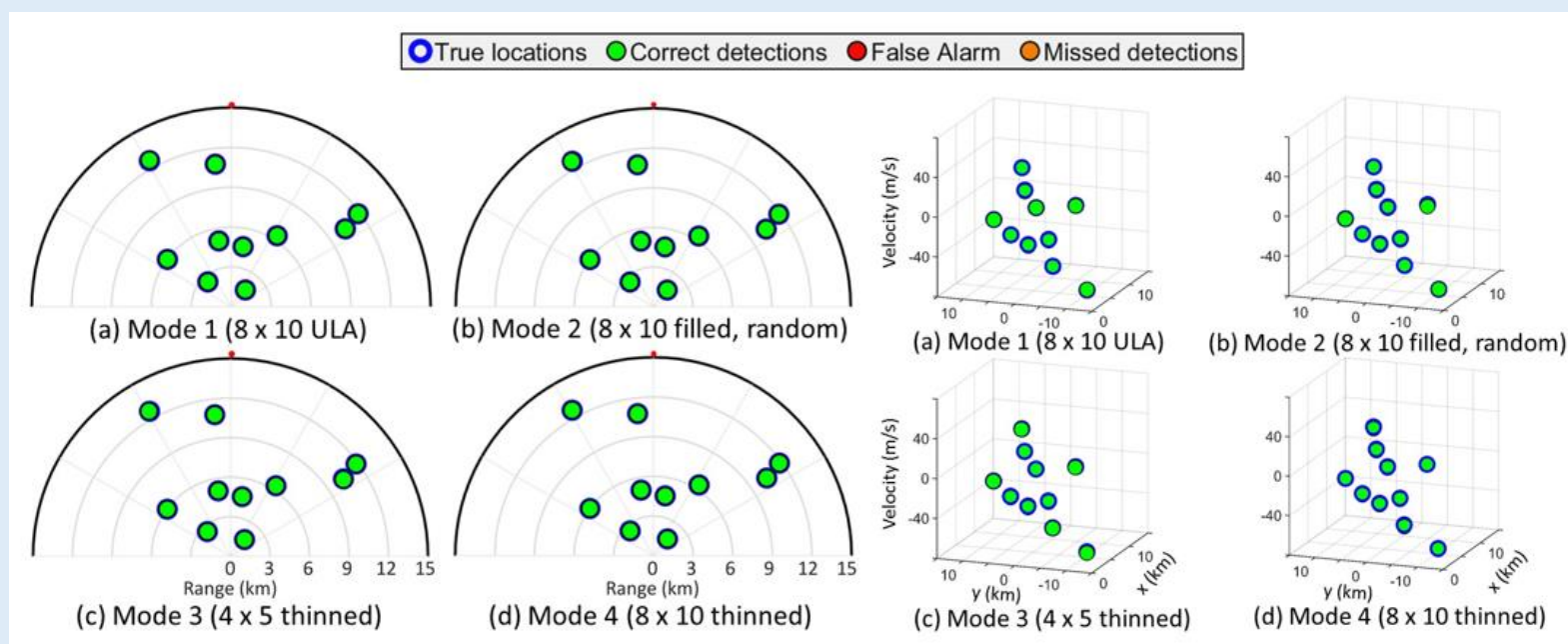


## User Interface \ Radar Display



## Sample Measurements Results

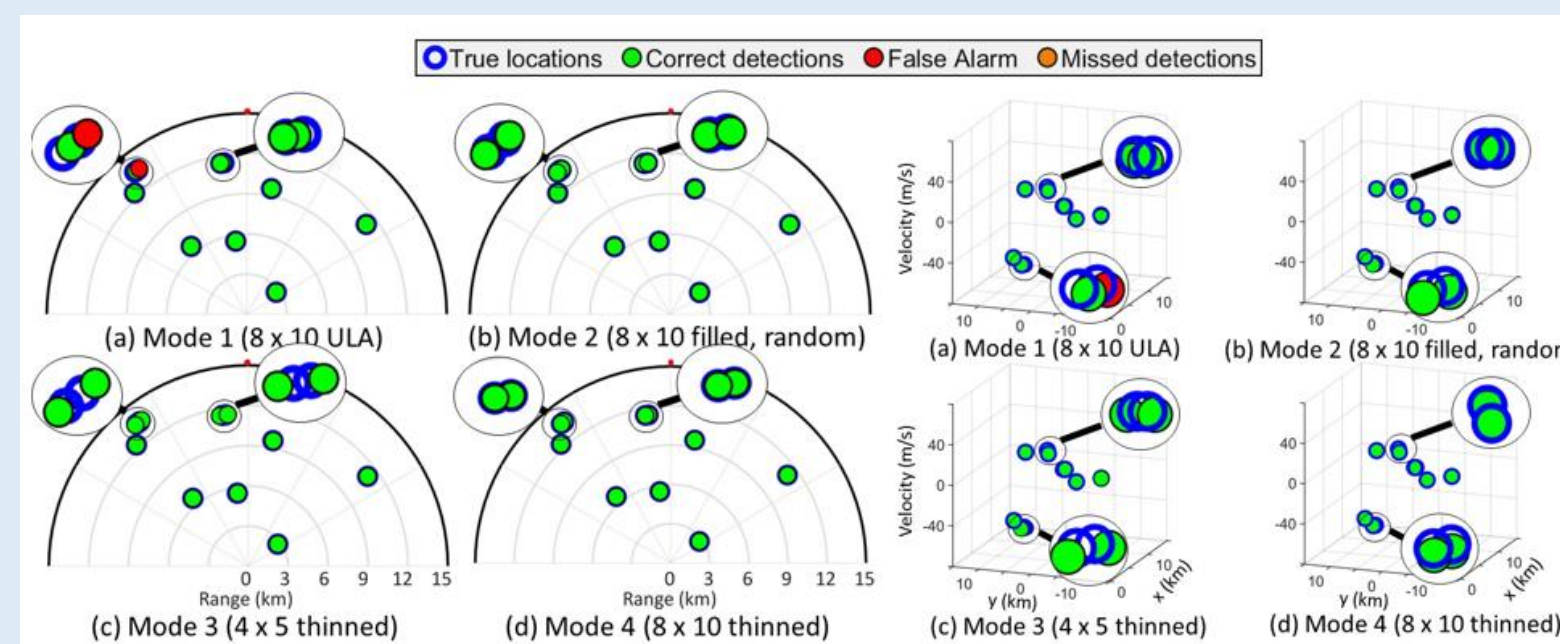
### Randomly Placed Targets



10 Targets	Min. angular spacing: 0.025	SNR = -15 dB
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Mode 3 (sub-Nyquist) detection performance is same as that of Mode 1 (Nyquist)

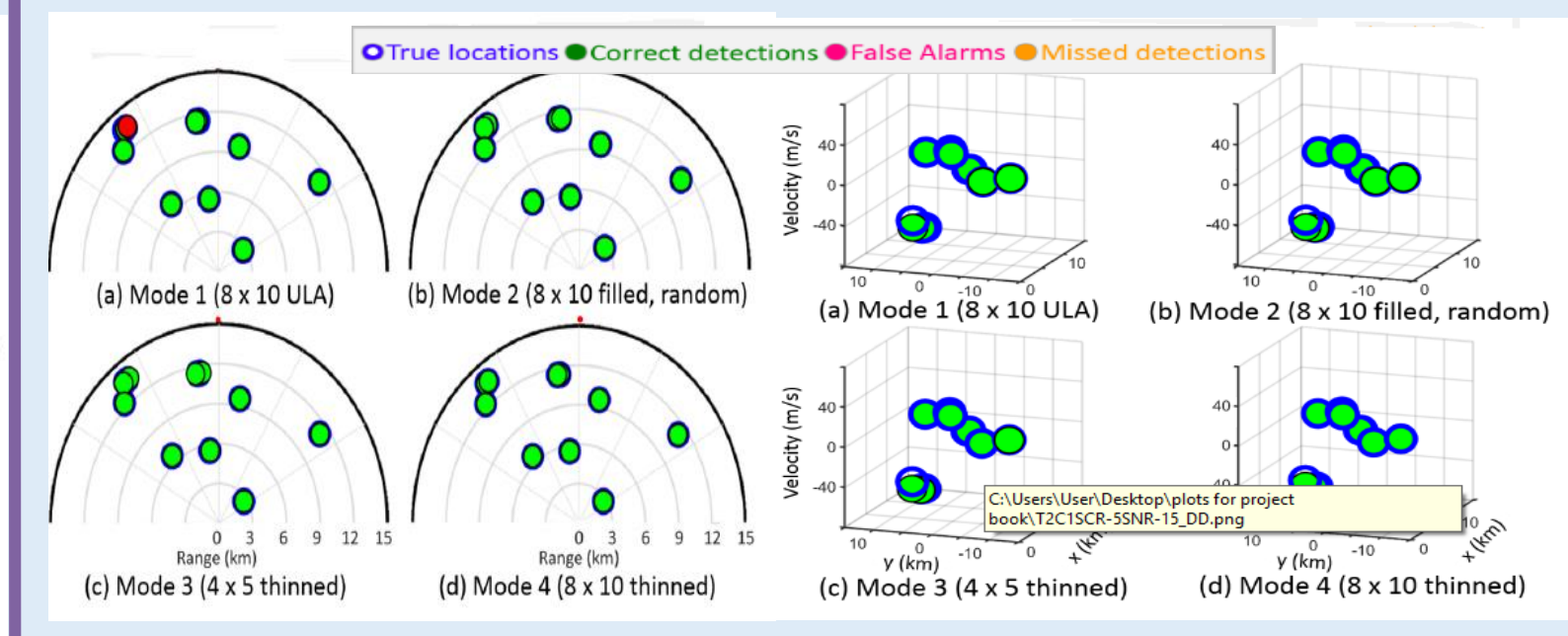
### Cognitive Sub-Nyquist Mode



10 Targets	Min. angular spacing: 0.02	SNR = -15 dB
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Cognitive mode 3 (sub-Nyquist 4x5) performs better than Nyquist in low SNR

### Clutter removal with 1 clutter cloud



10 Targets	Min. angular spacing: 0.02	SNR = -15 dB
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Cognitive mode 3 (sub-Nyquist 4x5) performs better than Nyquist in low SNR