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# Sub-Nyquist SAR via Fourier Domain Processing

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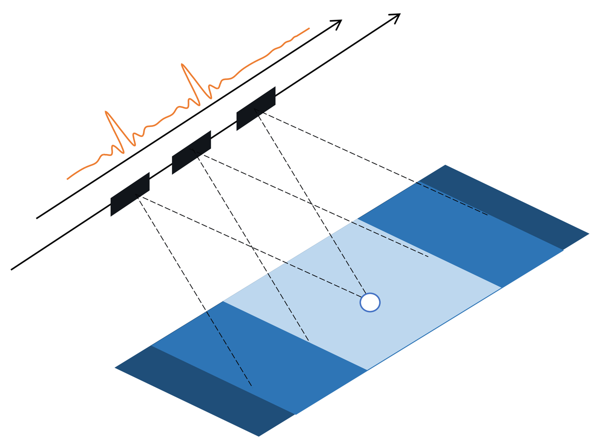
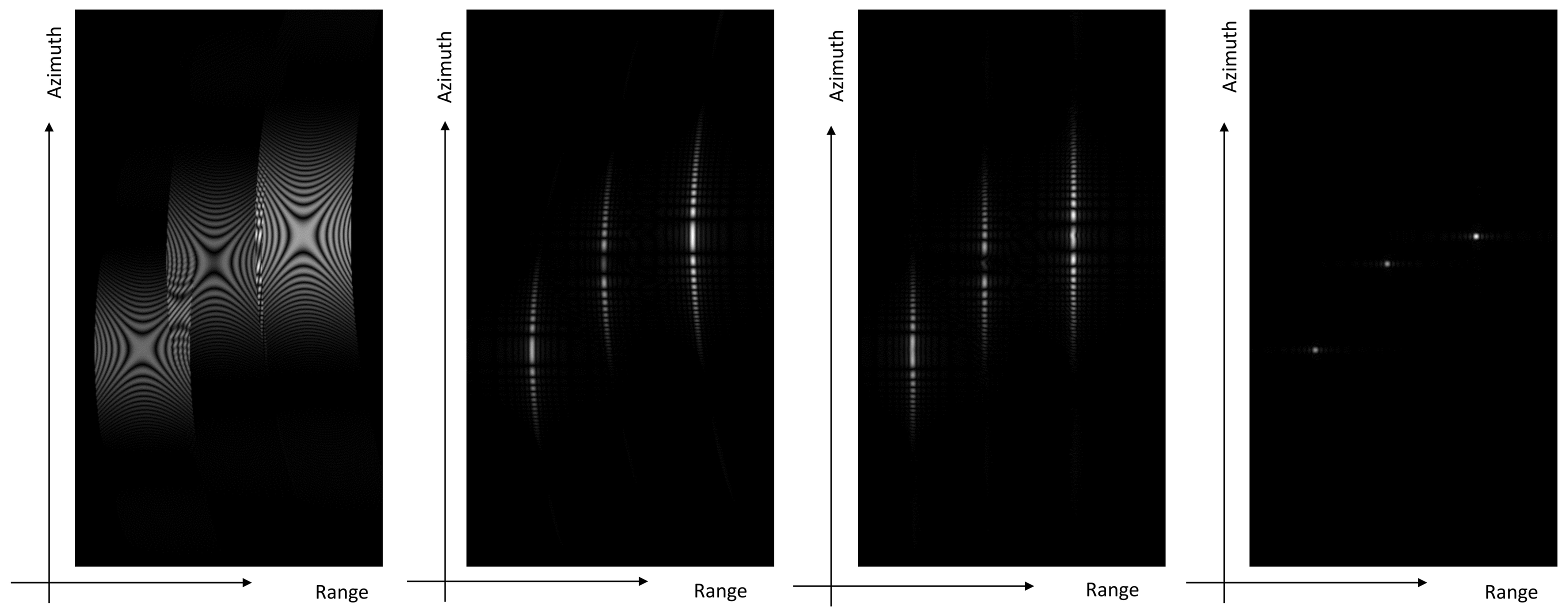
## Contributions

- **New SAR signal processing algorithm**, equivalent to the well-proved Range-Doppler Algorithm (RDA).
  - **Bypassing interpolation** in RDA.
  - **Avoiding over-sampling** which is used in practice.
  - **Enabling** a convenient framework for rate reduction.
- Reconstruction of SAR images using **Sub-Nyquist sampling** rates at the receiver.
  - Based on **Xampling** mechanism and **Compressive Sensing**.
  - Saving on board memory and overcoming downlink throughput requirements for orbital missions.
- **Fast 2D recovery** algorithm.
  - Exploiting 2D natural structure of image without the use of vectorization.
  - Fits to real data sets.

## Range-Doppler Processing

- Various algorithms have been developed in order to process the SAR received raw data, d[n,m], into an image. RDA is the most widely used approach for high resolution processing of SAR data.
- RDA contains three main stages: Range compression, Range Cell Migration Correction (RCMC), Azimuth compression.






Raw data → Range Compression → RCMC → Azimuth Compression

- The **RCMC** stage is aimed at decoupling the dependency between the azimuth and range axes and to correct the hyperbolic trajectory of the targets' echoes.
- The non-constant, non-integer shifts at the RCMC stage are realized by a digital subsequent interpolation which effectively increases the sampling rate of the system.

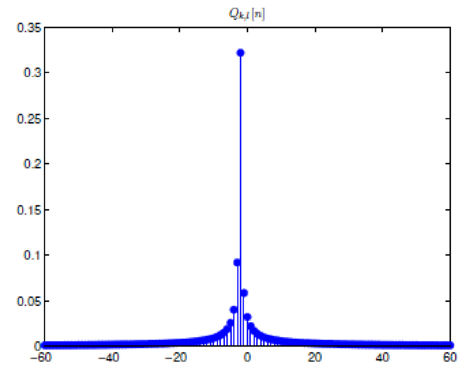
**Goal:** Getting rid of interpolation and reduce sampling rate at acquisition.

## Fourier Domain Range-Doppler

| | Conventional RDA | Fourier Domain RDA |
|---|---|---|
| Range Compression | $s[n,m]=d[n,m]*h^*[-n]$ | $\tilde{d}_m[l]=T\cdot d_m[l]*h^{s*}[l]$ |
| Azimuth DFT | $S[n,k]=\sum_{m=0}^{M-1}s[n,m]e^{\frac{-j2\pi km}{M}}$ | $s_k[l]=\sum_{m=0}^{M-1}\tilde{d}_m[l]e^{\frac{-j2\pi km}{M}}$ |
| RCMC | $\tilde{S}[n,k]=S[n+n\cdot ak^2,k]$ | $c_k[l]=\sum_{n\in\nu(k,l)}s_k[n]Q_{k,l}[-n]$ |
| Azimuth Compression | $Y[n,k]=\tilde{S}[n,k]e^{-j\pi\frac{k^2}{K_a[n]}}$ | $Y[n,k]=\left(\sum_{l=-N/2}^{N/2}c_k[l]e^{\frac{j2\pi nl}{T}}\right)\cdot\left(e^{-j\pi\frac{k^2}{K_a[n]}}\right)$ |
| Azimuth IDFT | $I[n,m]=\frac{1}{M}\sum_{k=0}^{M-1}Y[n,k]e^{\frac{j2\pi mk}{M}}$ | $I[n,m]=\frac{1}{M}\sum_{k=0}^{M-1}Y[n,k]e^{\frac{j2\pi mk}{M}}$ |

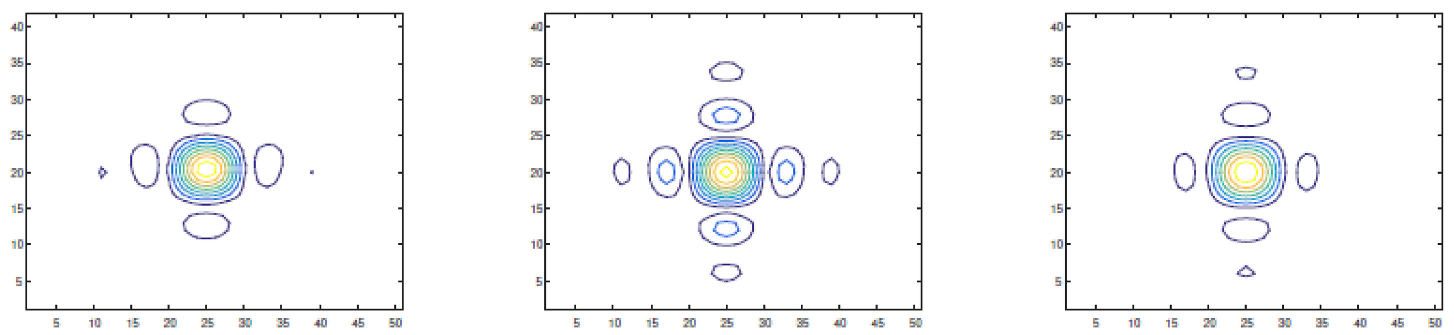
- Fourier domain RCMC is similar to Beamforming in frequency of ultrasound signals [1].

$$Q_{k,l}[n]=\frac{1}{1+ak^2}e^{-j\pi\left(n+\frac{l}{1+ak^2}\right)}sinc\left(n+\frac{l}{1+ak^2}\right)$$




**Interpolation is replaced by a weighting sum of Fourier coefficients, when the weight are characterized by a rapid decay.**

- Comparing SAR point spread function (PSF):

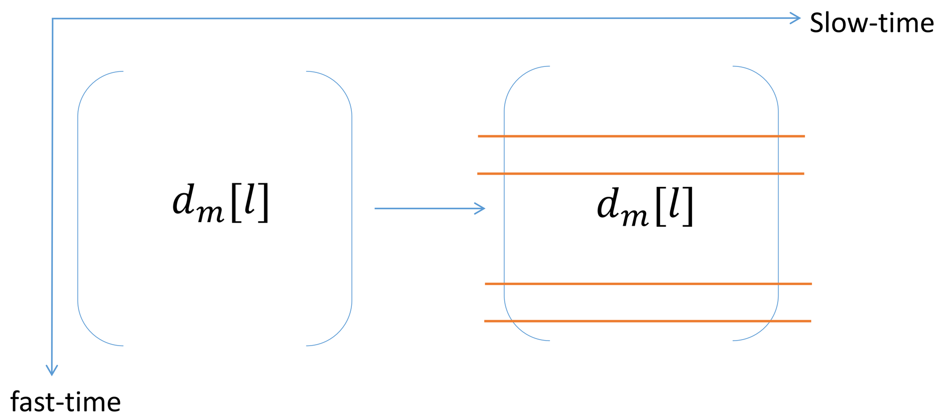


Conventional RDA | Fourier RDA, $|\nu(k,l)|=3$ | Fourier RDA, $|\nu(k,l)|=5$

**No over-sampling factor is required at the receiver**

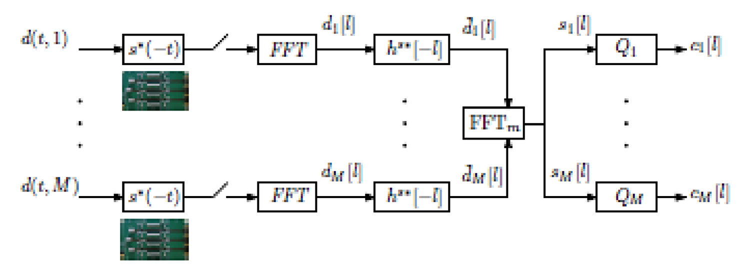
## Sub-Nyquist SAR

- The returned echoes are sampled in the Fourier domain under the Nyquist rate using Xampling [4]




Nyquist samples | Sub-Nyquist samples

**Preprocessing**




$$\{d_m[l]\}\to\{c_k[l]\}=C_p$$

## Fast 2D recovery

- Having the partial Fourier processed measurements, $C_p$, the image, $I$, is reconstructed by solving the optimization problem:

$$\min\|\Psi(I)\|_1 \; s.t \; \left\|C_p-F^s{}_p[B\circ(IF)]\right\|^2<\varepsilon$$

$F$ – DFT matrix
$F^S$ – Sampled Fourier series transformation
$B$ – Azimuth Compression matrix
$\Psi$ – Sparsifying transform

Recovery by extended FISTA [3]

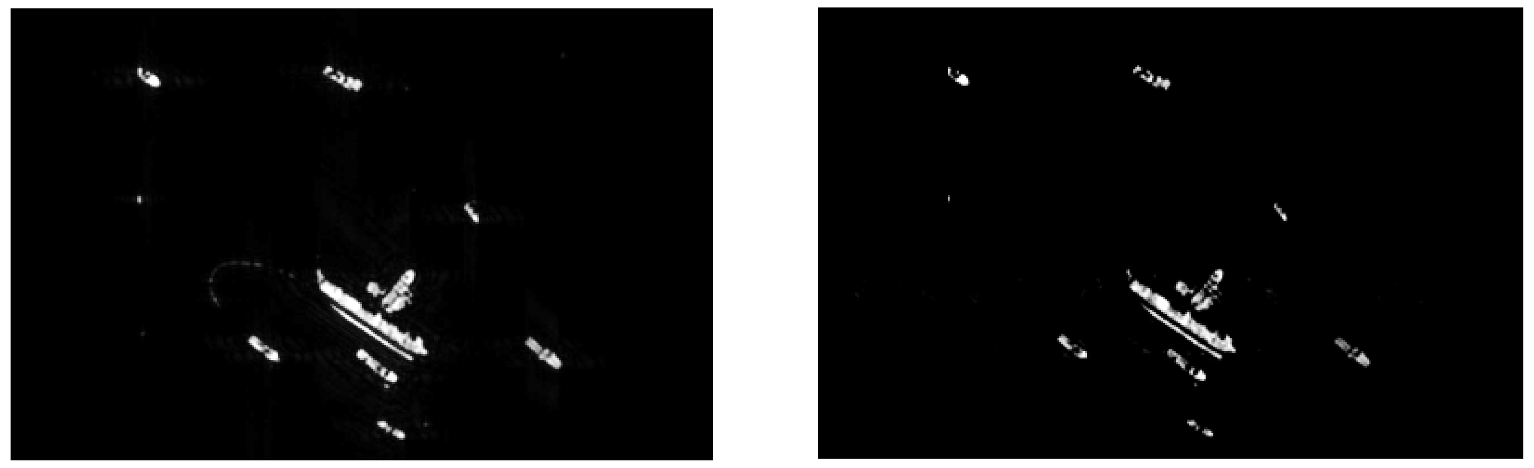
**Algorithm 1** SAR FISTA for sub-Nyquist sampling

**Input:** Xamples $\mathbf{D}_p=\{d_m[l]\}_{0\le m<M}^{l\in\tilde{\kappa}}$, measurement matrices $\mathbf{F}_p^s$, $\mathbf{B}$ and $\mathbf{F}$
**Output:** estimate for sparse coefficients of SAR image, $\hat{\mathbf{X}}$, such that $\mathbf{I}=\Psi^{-1}(\hat{\mathbf{X}})$

1: **Initialization:** $\mathbf{C}_p=\{c_k[l]\}_{0\le k<M}^{l\in\kappa}\leftarrow\mathbf{D}_p$ via (7), (8) and (9)
   **Initialize:** $\mathbf{X}^0=\mathbf{0}$, $\mathbf{X}^1=\mathbf{0}$, $t_0=1$, $t_1=1$, $k=1$ $\lambda_1,\beta\in(0,1)$, $\bar{\lambda}>0$
2: **while** not converged **do**
3: $\mathbf{Z}^k=\mathbf{X}^k+\frac{t_{k-1}-1}{t_k}\left(\mathbf{X}^k-\mathbf{X}^{k-1}\right)$
4: $\mathbf{U}^k=\mathbf{Z}^k-\frac{1}{L_f}\nabla\mathbf{F}\left(\Psi^{-1}(\hat{\mathbf{X}})\right)$, via (12)
5: $\mathbf{X}^{k+1}=\text{soft}\left(\mathbf{U}^k,\frac{\lambda_k}{L_f}\right)$
6: $t_{k+1}=\frac{1+\sqrt{4t_k^2+1}}{2}$
7: $\lambda_{k+1}=\max\left(\beta\lambda_k,\bar{\lambda}\right)$
8: $k=k+1$
9: **end while**
$\hat{\mathbf{X}}=\mathbf{X}$

- Spatially sparse scene, Ψ is the unit transform:

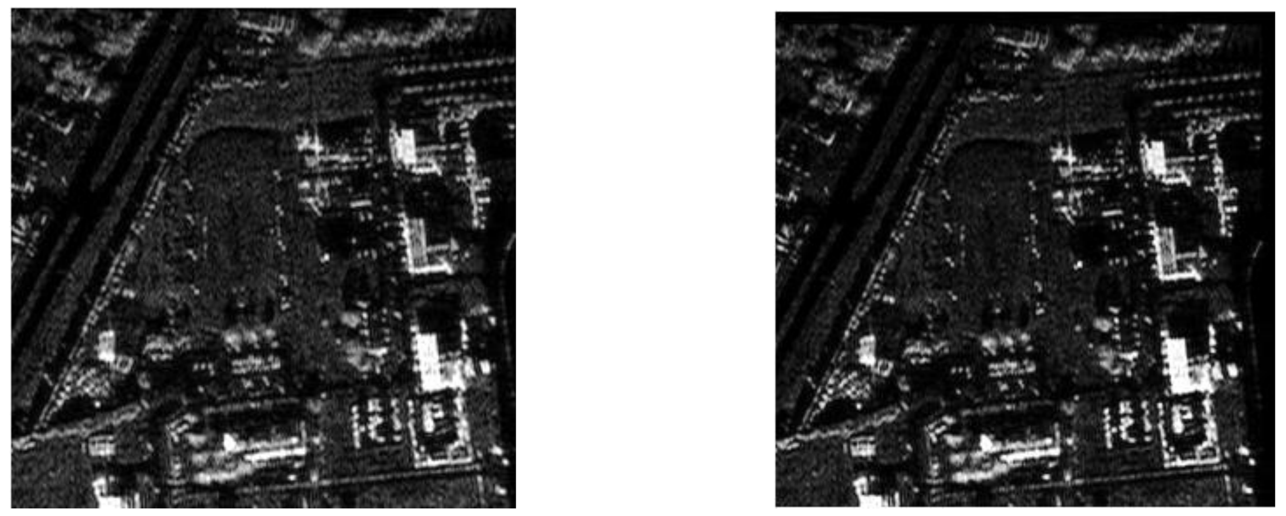
Conventional RDA | Fourier domain RDA Sub-Nyquist



**Using only 40% of Nyquist rate**

- Naturally sparse scene, Ψ is the wavelets transform:

Conventional RDA | Fourier domain RDA Sub-Nyquist



**Using only 45% of Nyquist rate**

## References

- [1] T. Chernyakova and Y. C. Eldar, "Fourier Domain Beamforming: The Path to Compressed Ultrasound Imaging", IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, 2014.
- [2] O. Bar – Ilan and Y. C. Eldar, "Sub-Nyquist radar via Doppler focusing", IEEE Transactions on Signal Processing, 2014.
- [3] A. Beck and M. Teboulle, "A fast iterative shrinkage-thresholding algorithm for linear inverse problems", SIAM journal on imaging sciences, 2009.
- [4] M. Mishali and Y. C. Eldar, "From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals," IEEE Journal of Selected Topics in Signal Processing, 2010