



$$\hat{x} = \arg \min_{x \in \mathbb{R}^n} \|x\|_1 + \lambda \|y - Ax\|_2^2$$

A Hardware Prototype of Sub-Nyquist Time-Based Sampling of FRI Signals

Hila Naaman, Satish Mulleti, Eliya Reznitskiy, Nimrod Glazer, Moshe Namer, and Yonina C. Eldar

Faculty of Math and CS, Weizmann Institute of Science, Rehovot, Israel

Email: hila.naaman@weizmann.ac.il

Motivation and Contributions

- Sampling and quantization are critical tasks of an ADC
- Conventional ADCs fail to utilize the signal information resulting in unnecessary power and bandwidth consumption
- Time encoding machines (TEM) utilize signal information and encode the input signal into a time sequence
- Increase amplitude noise robustness by moving the quantization process from the signal amplitude domain to the time domain
- We present a TEM hardware for power efficient sub-Nyquist sampling and recovery of finite-rate-of-innovation signals

FRI Sampling and Reconstruction

- Stream of known pulses: $x(t) = \sum_{\ell=1}^L a_{\ell} h(t - \tau_{\ell})$ (Known)
- Signal $x(t)$ is parametrized by amplitudes and time-delays
- $2L$ Fourier samples of $x(t)$ uniquely determine the parameters
- If the signal is defined on the interval $[0, T]$ the local the rate of innovation is $\frac{2L}{T}$
- Sub-Nyquist sampling scheme enables computation of the Fourier samples from low rate samples

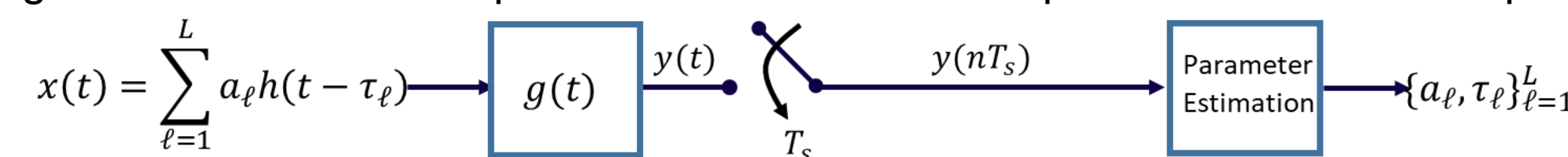
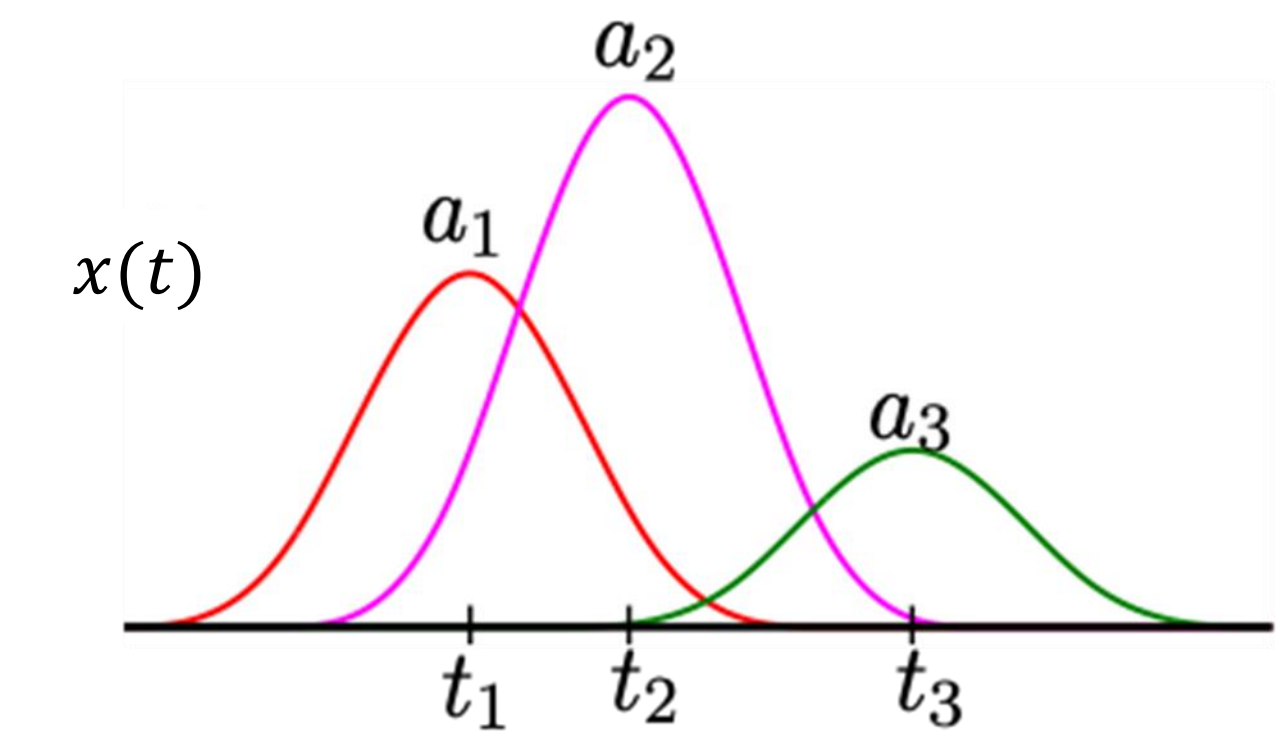
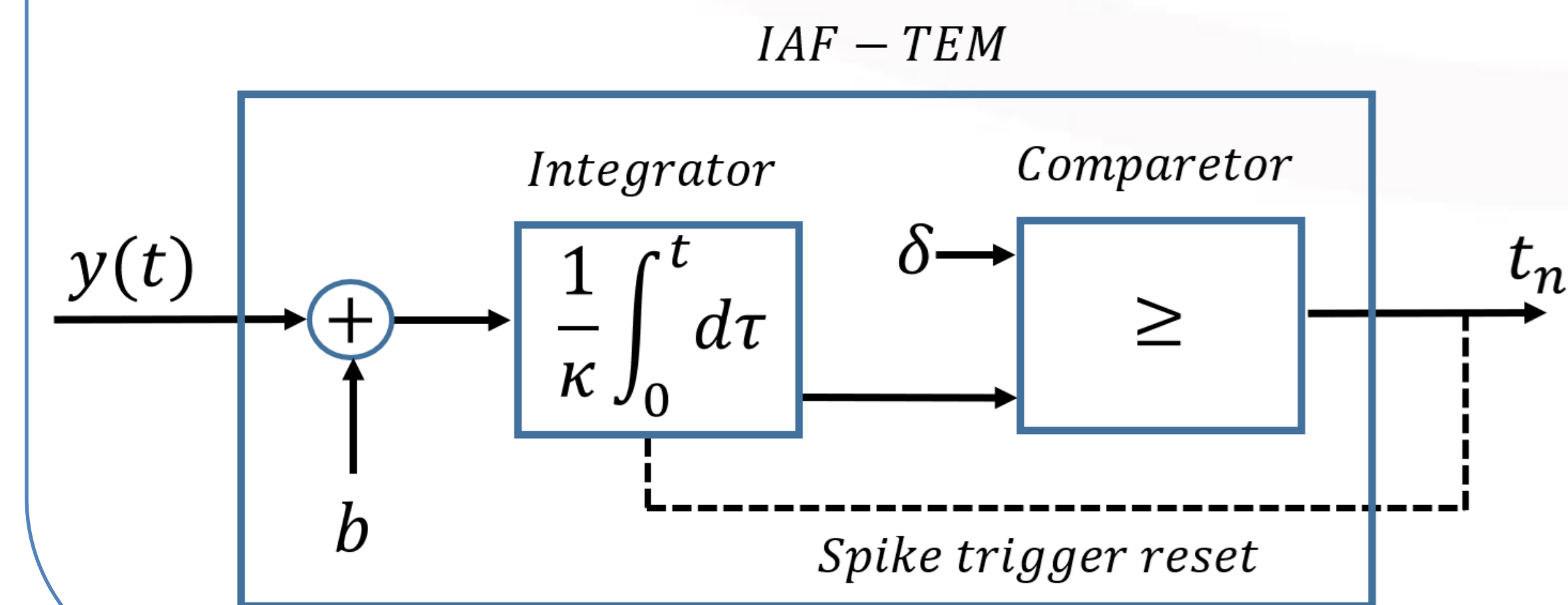


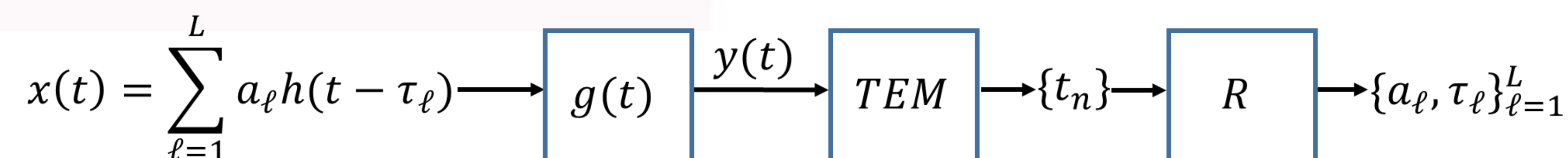
Fig: Sampling and reconstruction of FRI signal by using a sampling kernel $g(t)$

IAF-TEM

- An integrate-and-fire time-encoding machine is parameterised by:
 - b : The bias
 - κ : The integrator constant
 - δ : The threshold

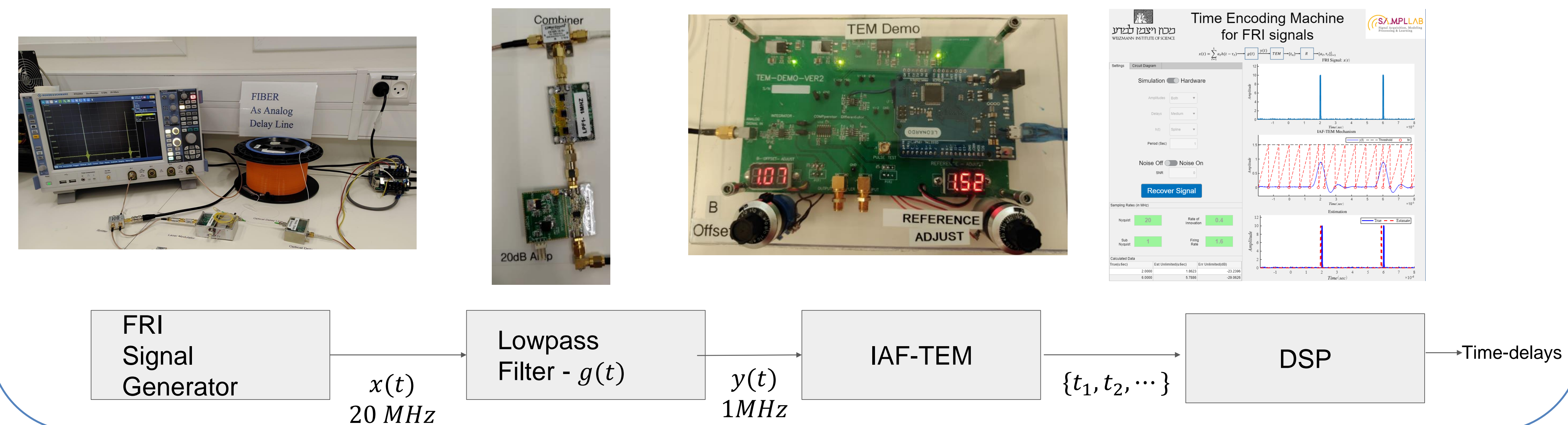


FRI-TEM Sampling

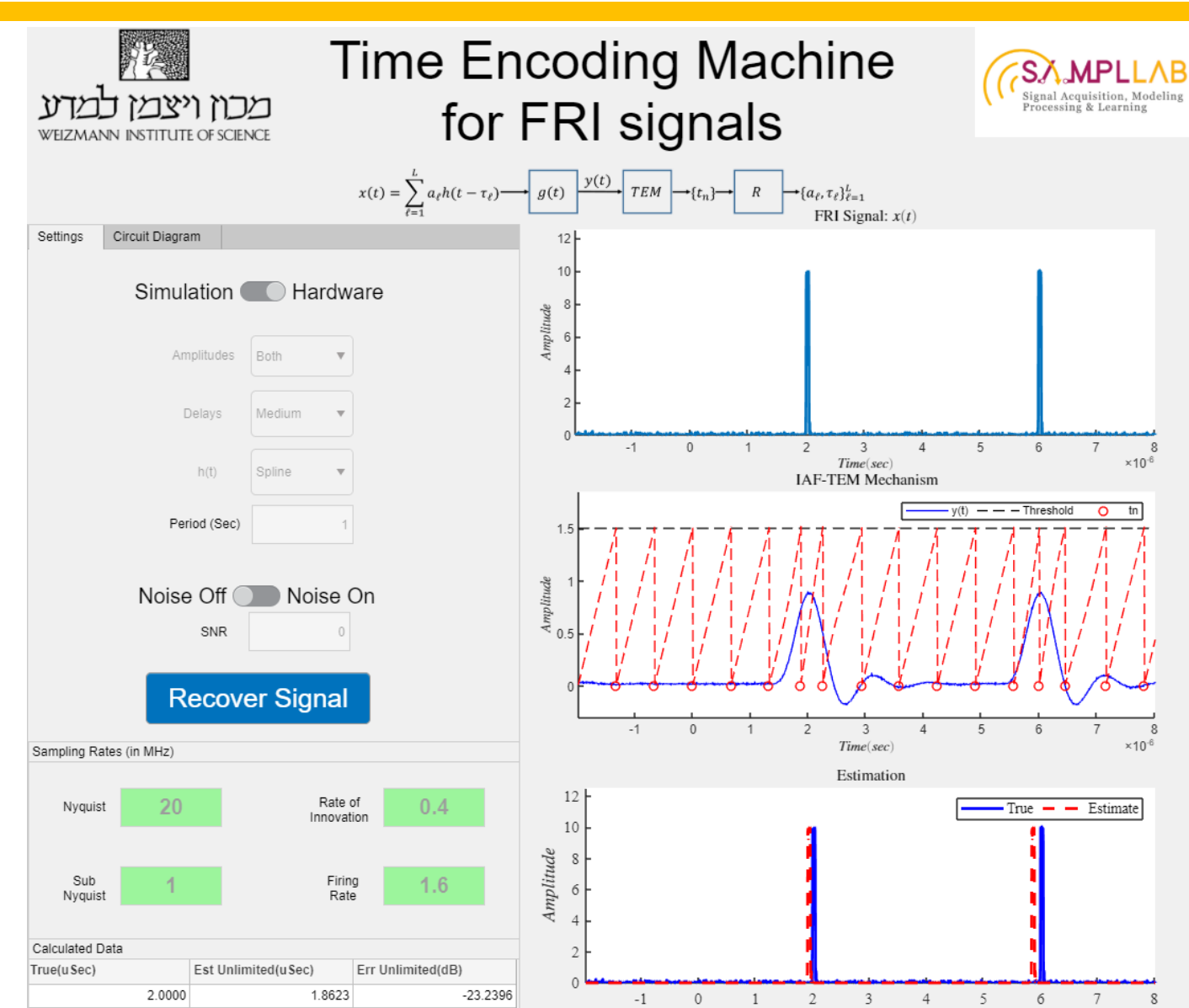


- We consider an FRI signal in the interval $[0, T]$
- The IAF-TEM input is a filtered FRI signal and the outputs are the time instants
- The IAF-TEM parameters are selected such that there are $2L + 2$ time instants within in a time interval T
- The IAF-TEM firing rate is defined as the number of time instants in the interval $[0, T]$
- Using TEM time instants $2L$ Fourier coefficients of $x(t)$ are computed from which FRI parameters are estimated

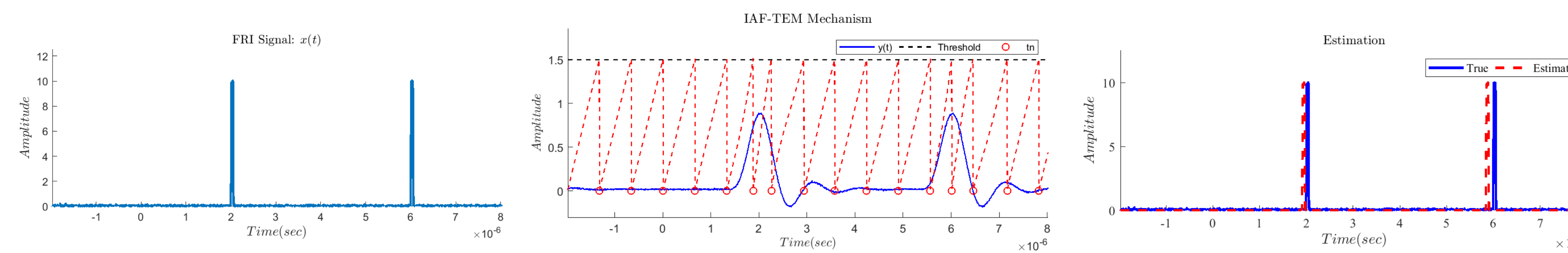
Hardware



User Interface



Results



Sampling rates: Nyquist 20MHz, Rate of innovation or sub-Nyquist: 0.4MHz,
TEM firing rate: 1.6MHz

- We consider an FRI signal $x(t)$ consisting of 2 pulses of bandwidth 20MHz and delay between them is 4μs
- The rate of innovation is 0.4MHz
- The signal is lowpass filtered with a cutoff frequency 0.5MHz
- The filtered signal $y(t)$ is passed through an IAF-TEM sampler resulting in firing a rate of 1.6MHz, $\sim \frac{1}{12}$ the Nyquist rate
- The FRI parameters are estimated up to -25dB SNR while operating at 12.5 times less than the Nyquist rate