# Sub-Nyquist Collocated MIMO Radar in Time and Space
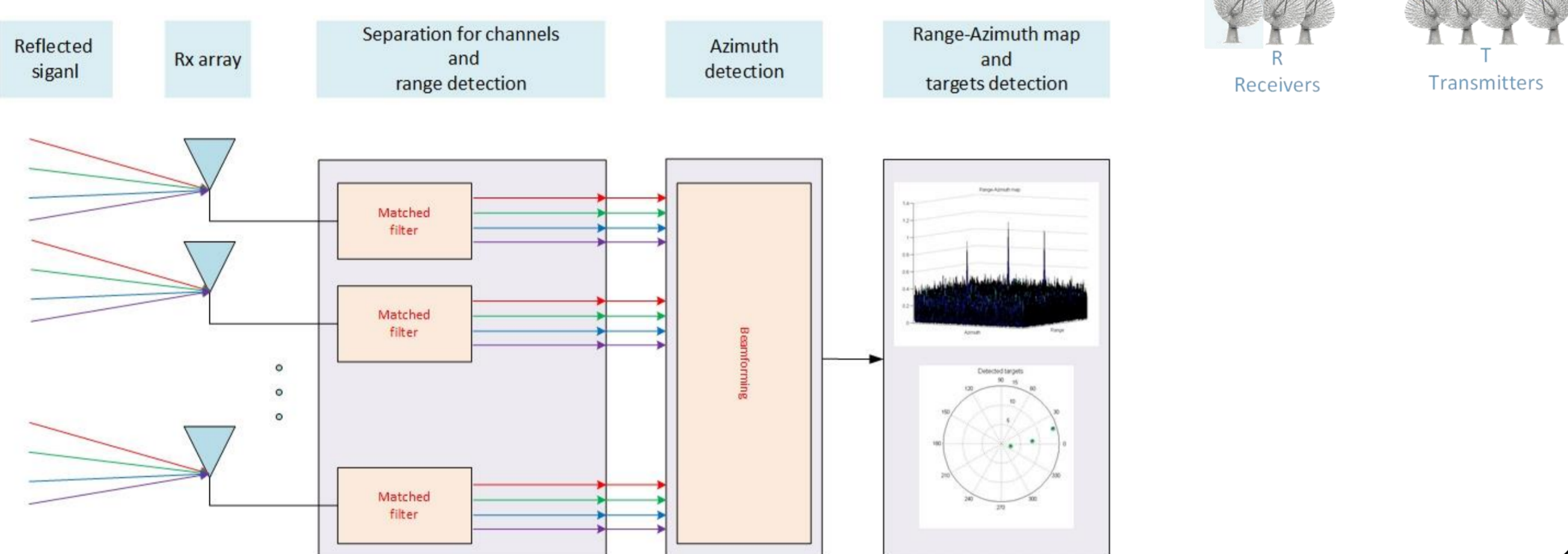
## David Cohen, Deborah Cohen and Yonina C. Eldar

## Contributions

- Application of the sub-Nyquist framework to collocated MIMO radar in time and space while preserving the range-azimuth resolution
  - Low rate sampling and digital processing
  - Reduced number of antennas
- Recovery algorithm scaled with problem size by adapting the OMP algorithm to matrix form
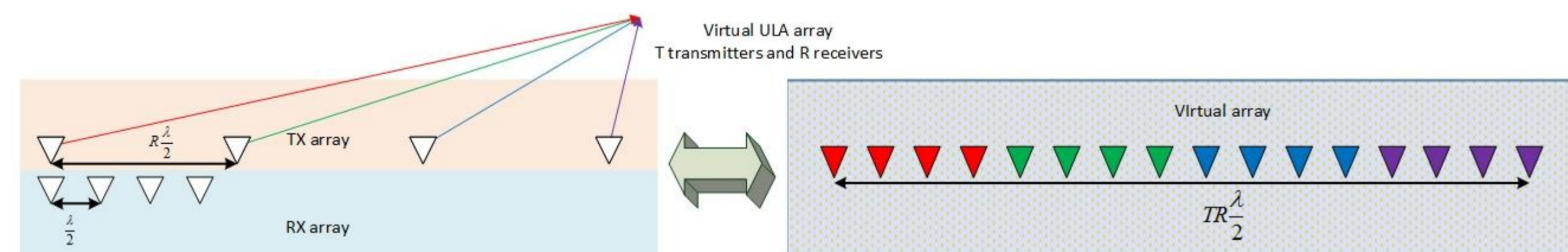
## Collocated MIMO Radar

- MIMO combines multiple transmit and receiver antenna elements
- Each transmitting element radiates orthogonal waveforms
- **Core idea:** achieving high spatial resolution by separation and coherent processing of the receivers' channels
- All space is uniformly lit - beamforming is done at the receiver
- **Conventional processing for range-azimuth recovery:**



## Resolution in Time and Space

- Classic approach adopts a virtual ULA structure



**Resolution in the space domain**
- Azimuth resolution: determined by virtual ULA's aperture
- **Problem:**
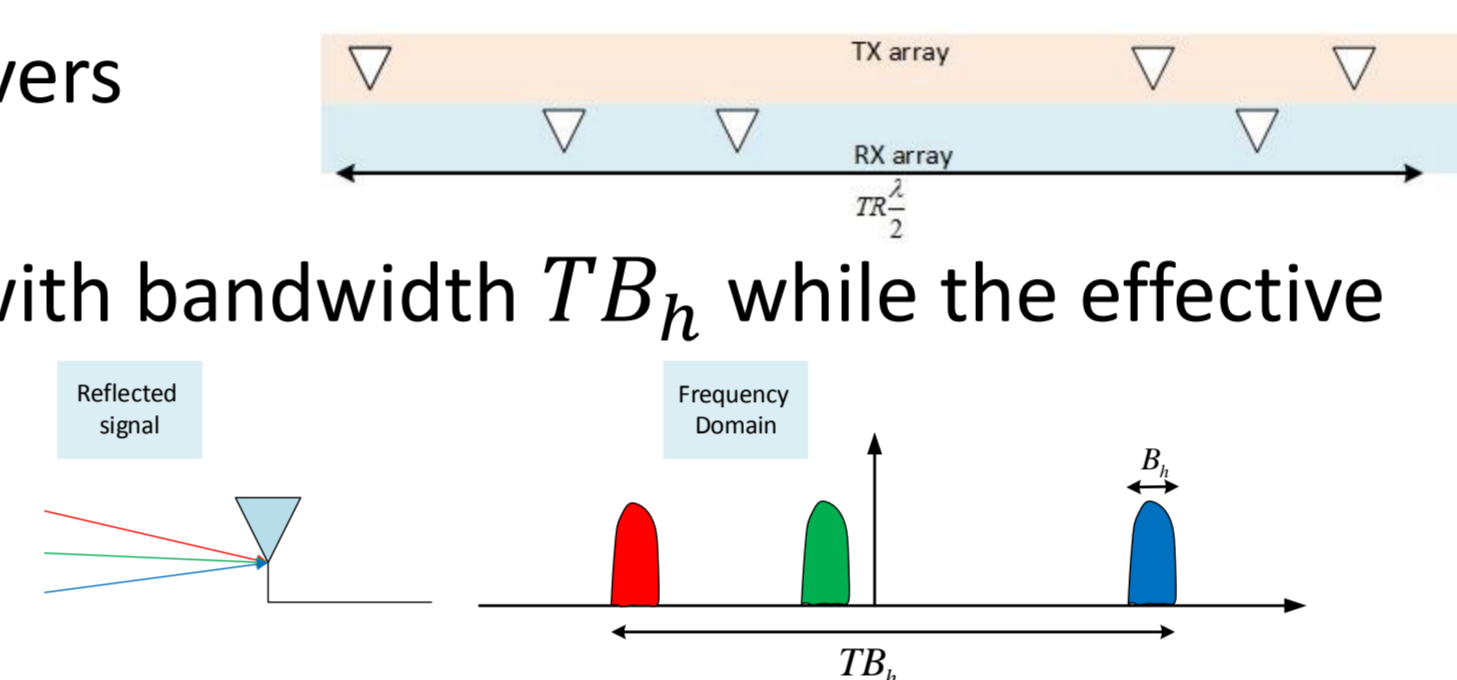  High resolution in the azimuth ⟷ large virtual ULA's aperture requires large number of elements

**Resolution in the time domain**
- Range resolution: determined by transmitted signal's bandwidth
- **Problem:**
  High resolution in the range ⟷ large signal bandwidth requires high sampling rate

Goal: break link between number of elements and spatial resolution and sampling rate and time resolution
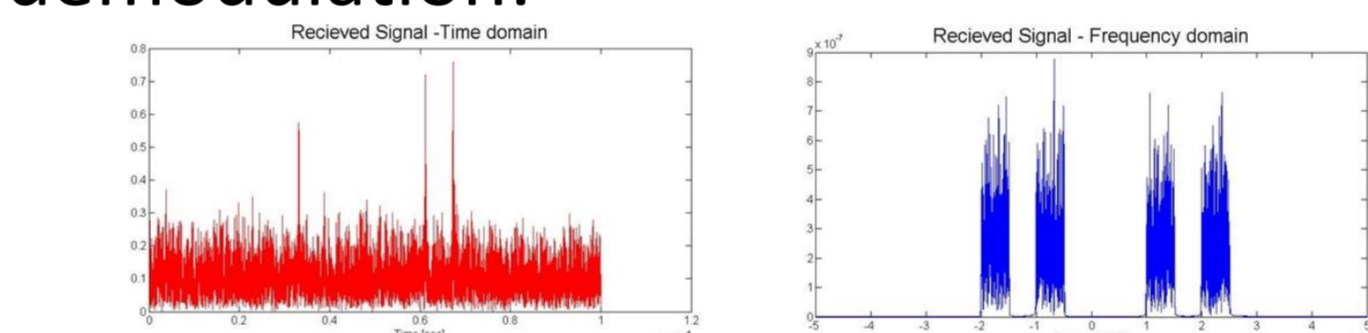
## Proposed Array Structure and Signal Model

- Frequency division approach
- Random array

**Sub-Nyquist in space**
- Preserving azimuth resolution of T transmitters and R receivers while using M<T transmitters and Q<R receivers
- Elements are randomly located with uniform distribution across the virtual ULA's aperture of T transmitters and R receivers



**Sub-Nyquist in time**
- Preserving range resolution of signal with bandwidth $TB_h$ while the effective sampling rate is lower
- The transmissions are performed over total bandwidth $TB_h$.
- For each transmission, sub-Nyquist sampling scheme is applied



## Xampling in Time and Space

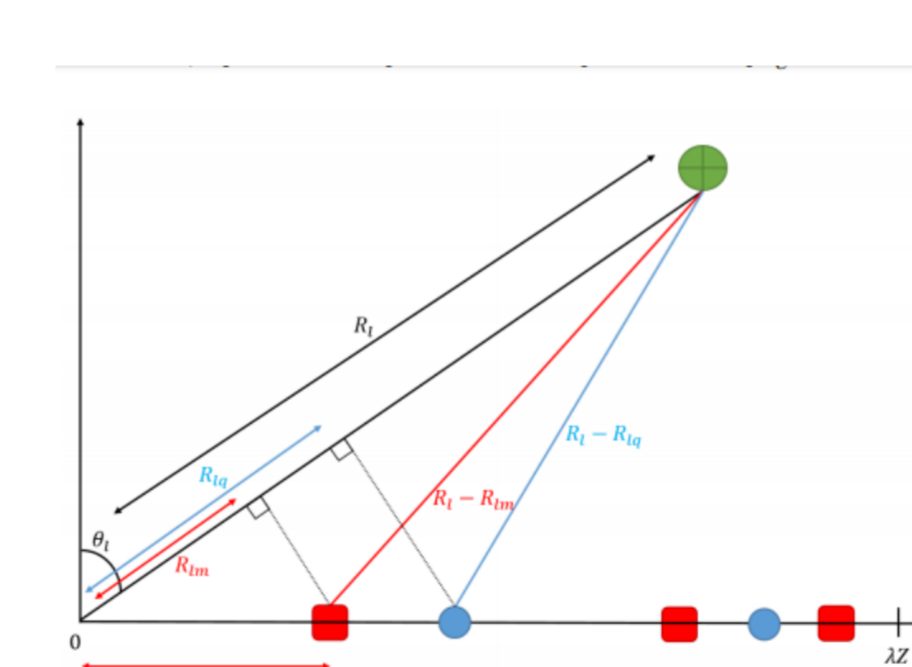- Received signal at the $q$th antenna after demodulation:

$$x_q(t) = \sum_{m=0}^{M-1} \sum_{l=1}^{L} h_m(t - \tau_l) e^{j2\pi\beta_{mq}\vartheta_l}$$



- Goal: estimate the targets range and azimuth $\tau_l, \vartheta_l$.

- Fourier coefficients of the channel between $m$th transmitter and $q$th receiver:

$$y_{m,q}[k] = \sum_{l=1}^{L} \alpha_l e^{j2\pi\beta_{mq}\vartheta_l} e^{-j\frac{2\pi}{\tau}k\tau_l} e^{-j2\pi f_m \tau_l}$$
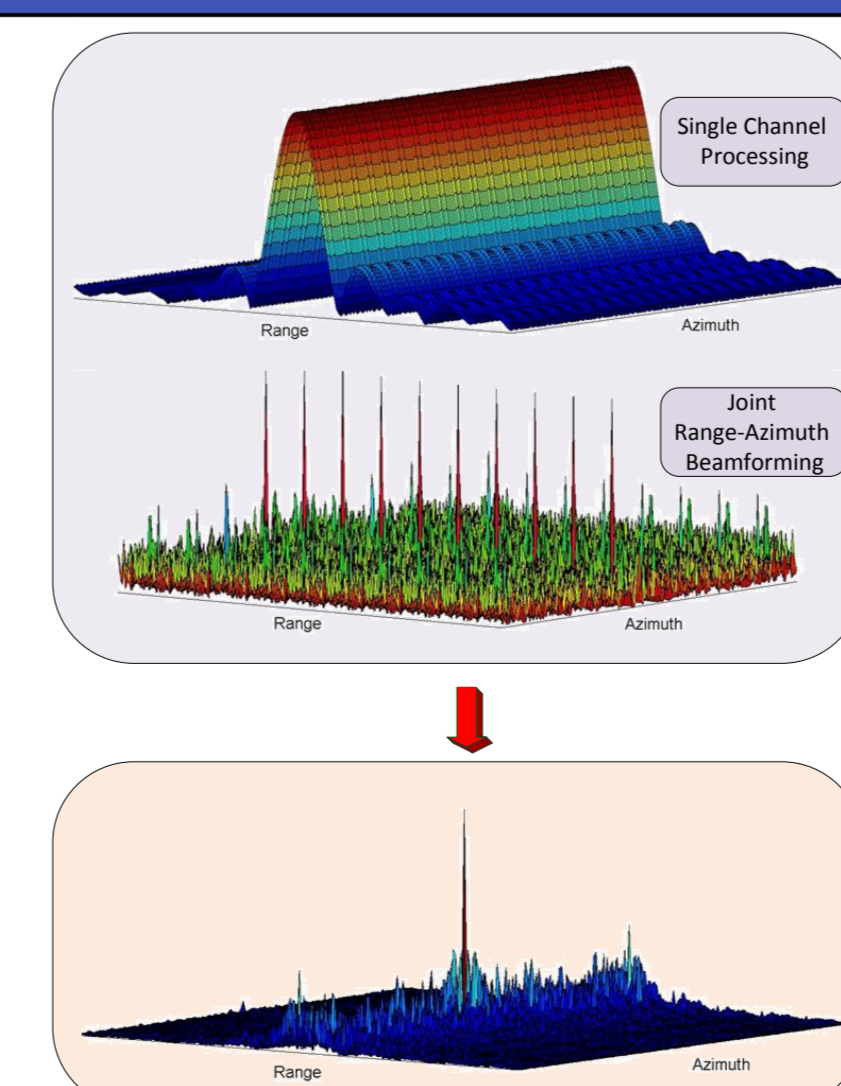


- $\beta_{mq}$ is governed by the elements location while $f_m$ by the carriers frequencies
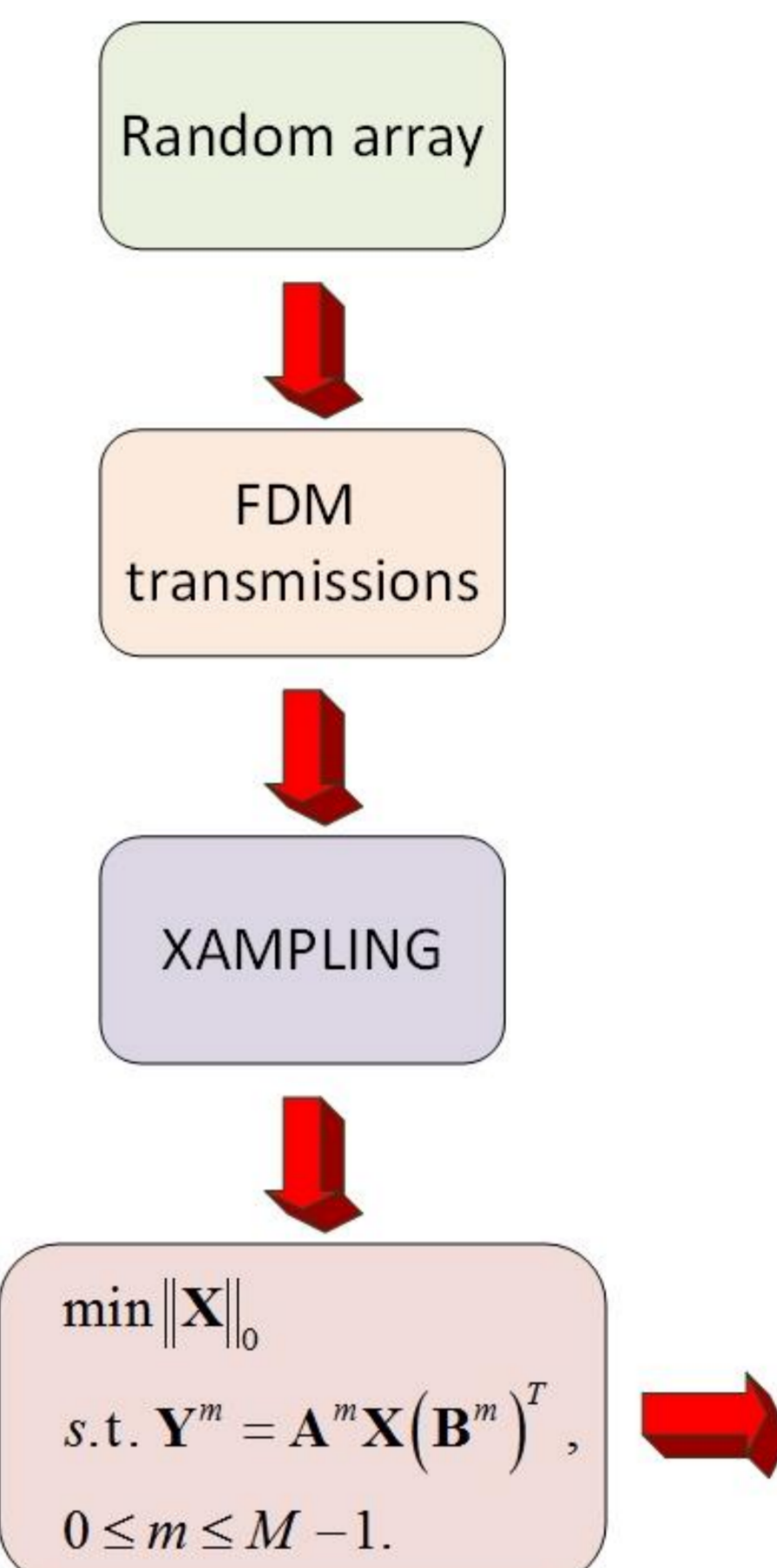- Xampling: obtain set of Fourier coefficients from low rate samples

- Fourier coefficients for the $m$th transmission in matrix form $\mathbf{Y}^m = \mathbf{A}^m \mathbf{X}(\mathbf{B}^m)^T$
  - $\mathbf{A}^m, \mathbf{B}^m$: range-azimuth dictionaries
  - $\mathbf{X}$: sparse matrix whose elements are located at the targets' range-azimuth

## Joint Range-Azimuth Detection

- Frequency diversity: channels are azimuth-dependent and range dependent

- Range-azimuth coupling resolved by using random array and joint range-azimuth detection

- By processing all channels together: achieve range resolution according to total bandwidth



## Recovery algorithm via OMP Matrix Approach



$$\min \|\mathbf{X}\|_0$$
$$s.t. \ \mathbf{Y}^m = \mathbf{A}^m \mathbf{X}(\mathbf{B}^m)^T,$$
$$0 \le m \le M-1.$$

**Algorithm 1** OMP for simultaneous sparse matrix recovery
**Input:** observation matrices $\mathbf{Y}^m$, measurement matrices $\mathbf{A}^m, \mathbf{B}^m$, for all $0 \le m \le M-1$
**Output:** index set $\Lambda$ containing the locations of the non zero indices of $\mathbf{X}$, estimate for sparse matrix $\hat{\mathbf{X}}$
1: Initialization: residual $\mathbf{R}_0^m = \mathbf{Y}^m$, index set $\Lambda_0 = \emptyset$, $t = 0$
2: Project residual onto measurement matrices:
$$\Psi = \mathbf{A}^H \mathbf{R} \bar{\mathbf{B}}$$
where $\mathbf{A} = [\mathbf{A}^{0^T} \mathbf{A}^{1^T} \cdots \mathbf{A}^{(M-1)^T}]^T$ and $\mathbf{B} = [\mathbf{B}^{0^T} \mathbf{B}^{1^T} \cdots \mathbf{B}^{(M-1)^T}]^T$ and $\mathbf{R} = \text{diag}([\mathbf{R}_t^0 \cdots \mathbf{R}_t^{M-1}])$ is block diagonal
3: Find the two indices $\lambda_t = [\lambda_t(1) \quad \lambda_t(2)]$ such that
$$[\lambda_t(1) \quad \lambda_t(2)] = \arg\max_{i,j} |\Psi_{i,j}|$$
4: Augment index set $\Lambda_t = \Lambda_t \bigcup\{\lambda_t\}$
5: Find the new signal estimate
$$\alpha = [\alpha_1 \dots \alpha_t]^T = (\mathbf{D}_t^T \mathbf{D}_t)^{-1} \mathbf{D}_t^T \text{vec}(\mathbf{Y})$$
6: Compute new residual
$$\mathbf{R}_t^m = \mathbf{Y}^m - \sum_{i=1}^{t} \alpha_i \mathbf{a}_{\lambda_t(l,1)}^m (\mathbf{b}_{\Lambda_t(l,2)}^m)^T$$
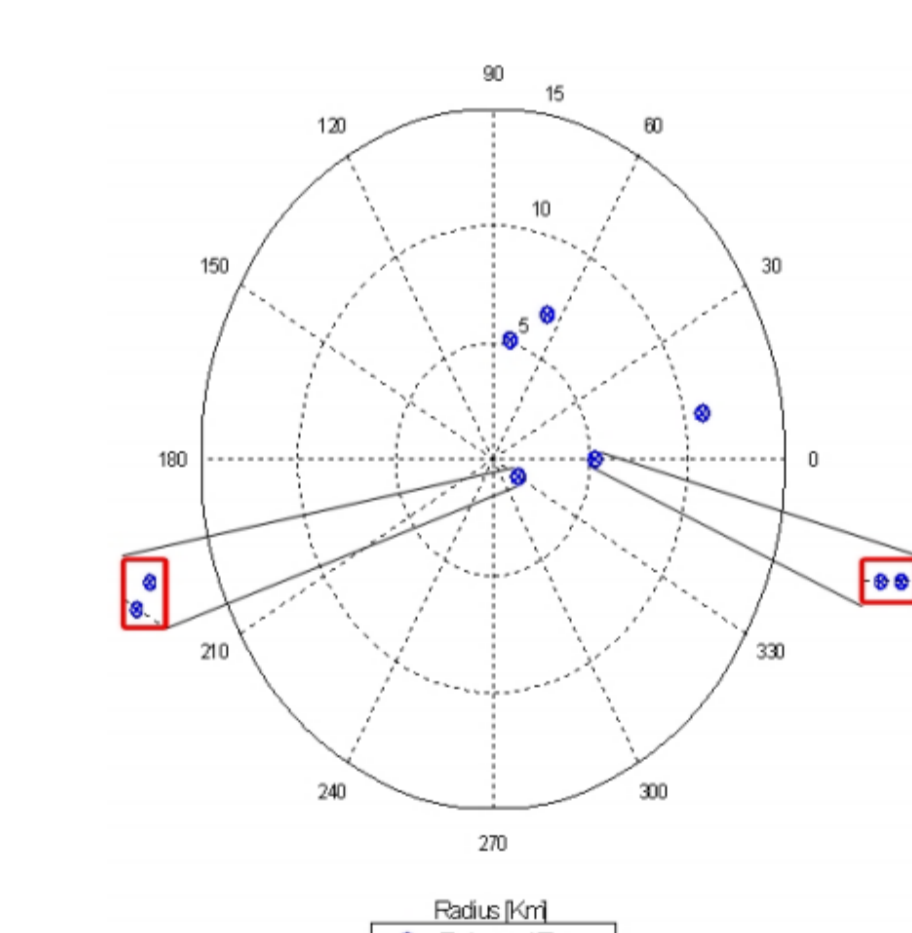7: Increment $t$ and return to step 2 if $t \le L$, otherwise stop
8: Estimated support set $\hat{\Lambda} = \Lambda_L$
9: Estimated matrix $\hat{\mathbf{X}}$: $(\Lambda_L(l,1), \Lambda_L(l,2))$-th component of $\hat{\mathbf{X}}$ is given by $\alpha_l$ for $l = 1, \cdots, L$ while rest of the elements are zero

## Simulation results

**Graph 1: Resolution in range and Azimuth**
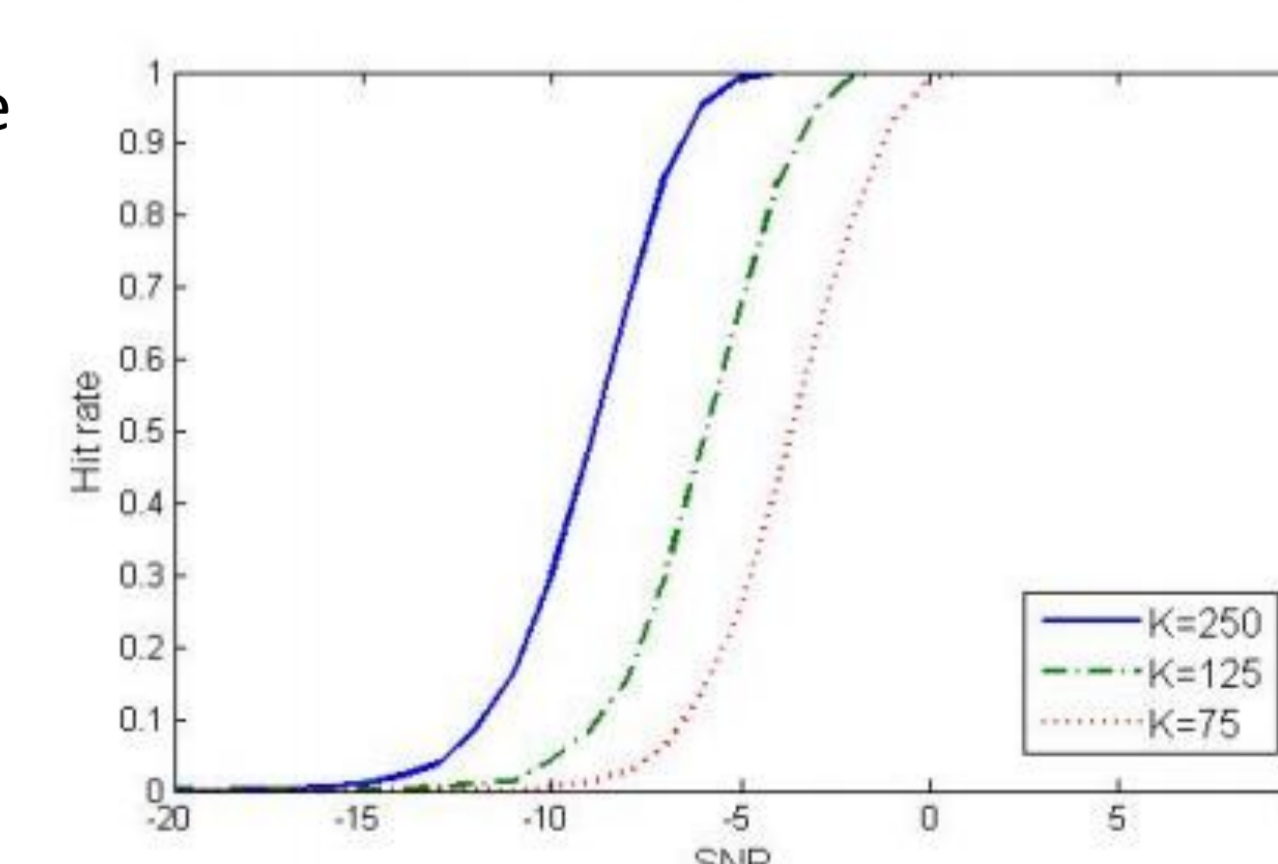7 targets including couple of targets with close range and a couple with close azimuth

| | | | |
|---|---|---|---|
| **Spatial Reduction** | T=20 | M=10 | 50% |
| | R=20 | Q=10 | 50% |
| **Time Reduction** | N=500 | K=250 | 50% |

**Total samples reduction: 12.5%**



**Graph 2: Hit rate vs SNR**
10 targets range-azimuth recovery performance

| | | | |
|---|---|---|---|
| **Spatial Reduction** | T=20 | M=10 | 50% |
| | R=20 | Q=10 | 50% |
| **Time Reduction** | N=500 | K=250 | 50% |
| | | K=125 | 25% |
| | | K=75 | 15% |

**Total samples reduction: 3.75%**



## References

- [1] M. Mishali and Y. C. Eldar, "From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals", *IEEE Journal of Selected Topics in Signal Processing*, 2010.
- [2] J. LI and P. Stoica, "MIMO radar with colocated antennas", *IEEE Signal Processing Magazine, 2007.*
- [3] M. Rossi A.M. Haimovich and Y. C. Eldar, "Spatial compressive sensing for MIMO radar", *Signal Processing IEEE Transactions on,* 2014.