Sub-Nyquist Collocated MIMO Radar in Time and Space

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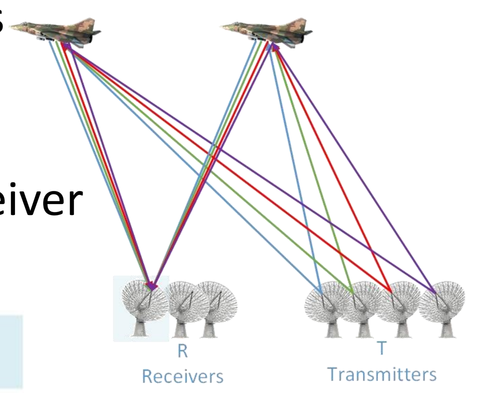
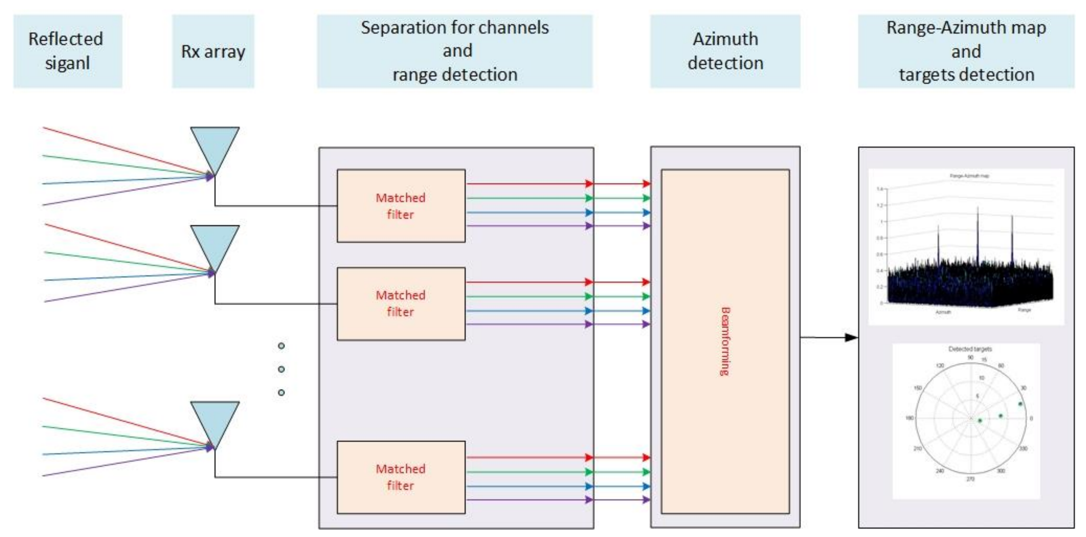
Contributions

- Application of the sub-Nyquist framework to collocated MIMO radar in time and space while preserving the range-azimuth resolution
  - Low rate sampling and digital processing
  - Reduced number of antennas
- Recovery algorithm scaled with problem size by adapting the OMP algorithm to matrix form

Collocated MIMO Radar

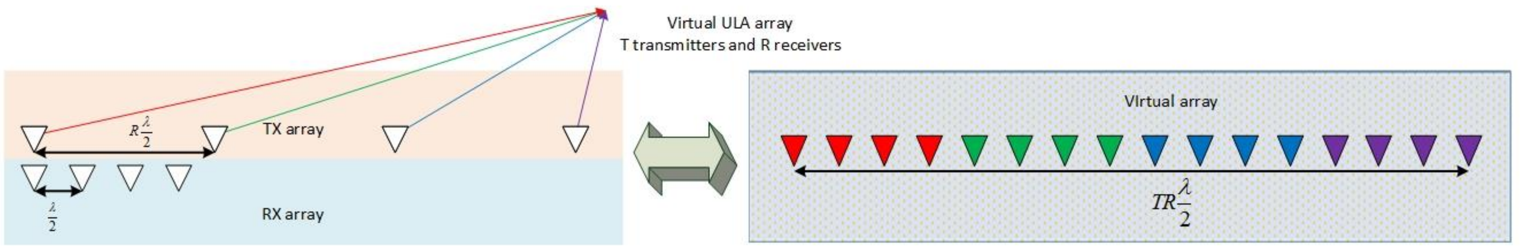
- MIMO combines multiple transmit and receiver antenna elements
- Each transmitting element radiates orthogonal waveforms
- **Core idea:** achieving high spatial resolution by separation and coherent processing of the receivers' channels
- All space is uniformly lit - beamforming is done at the receiver
- **Conventional processing for range-azimuth recovery:**





Resolution in Time and Space

- Classic approach adopts a virtual ULA structure



Resolution in the space domain

- Azimuth resolution: determined by virtual ULA's aperture
- **Problem:**

High resolution in the azimuth ⟷ virtual ULA's aperture
➞ requires large number of elements

Resolution in the time domain

- Range resolution: determined by transmitted signal's bandwidth
- **Problem:**

High resolution in the range ⟷ large signal bandwidth
➞ requires high sampling rate

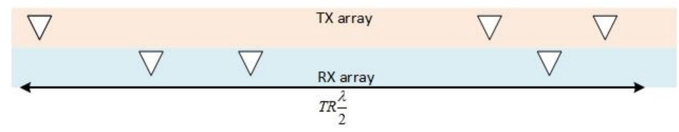
Goal: break link between number of elements and spatial resolution and sampling rate and time resolution

Proposed Array Structure and Signal Model

- Frequency division approach
- Random array

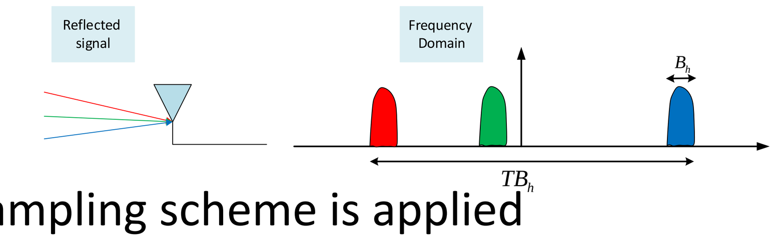
Sub-Nyquist in space

- Preserving azimuth resolution of T transmitters and R receivers while using M<T transmitters and Q<R receivers
- Elements are randomly located with uniform distribution across the virtual ULA's aperture of T transmitters and R receivers



Sub-Nyquist in time

- Preserving range resolution of signal with bandwidth TB_h while the effective sampling rate is lower
- The transmissions are performed over total bandwidth TB_h.

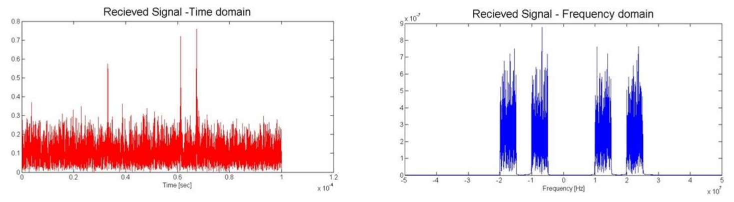


- For each transmission, sub-Nyquist sampling scheme is applied

Xampling in Time and Space

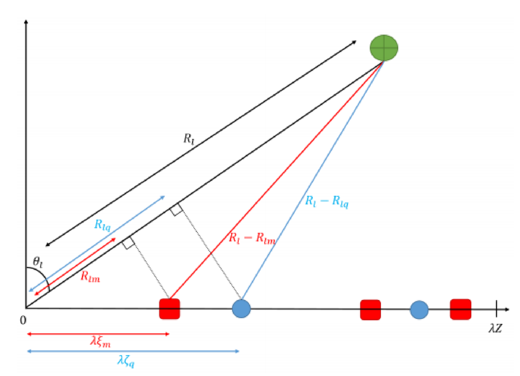
- Received signal at the qth antenna after demodulation:

$$x_q(t)=\sum_{m=0}^{M-1}\sum_{l=1}^{L}h_m(t-\tau_l)e^{j2\pi\beta_{mq}\vartheta_l}$$



- Goal: estimate the targets range and azimuth $\tau_l,\ \vartheta_l$.
- Fourier coefficients of the channel between mth transmitter and qth receiver:

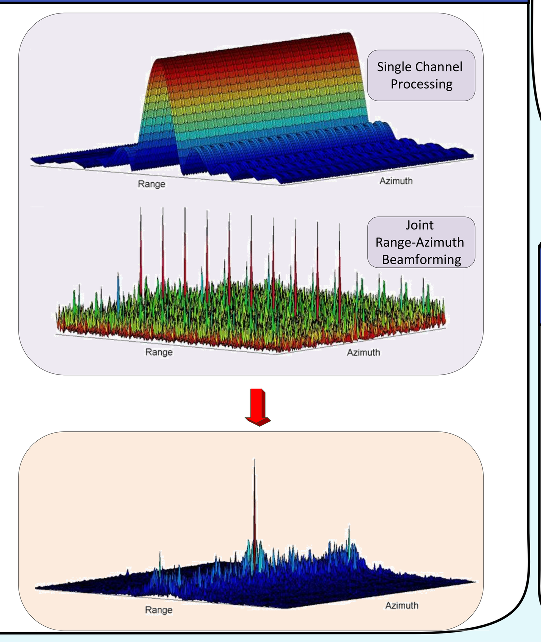
$$y_{m,q}[k]=\sum_{l=1}^{L}\alpha_l e^{j2\pi\beta_{mq}\vartheta_l}e^{-j\frac{2\pi}{\tau}k\tau_l}e^{-j2\pi f_m\tau_l}$$



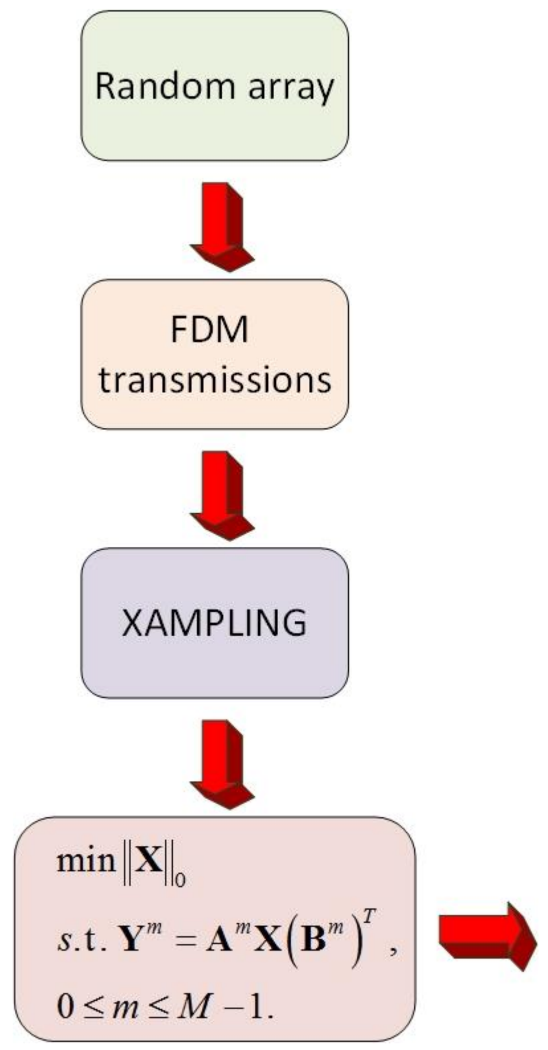
- β_{mq} is governed by the elements location while f_m by the carriers frequencies
- Xampling: obtain set of Fourier coefficients from low rate samples
- Fourier coefficients for the mth transmission in matrix form $\mathbf{Y}^m=\mathbf{A}^m\mathbf{X}\left(\mathbf{B}^m\right)^T$
  - $\mathbf{A}^m,\mathbf{B}^m$: range-azimuth dictionaries
  - $\mathbf{X}$: sparse matrix whose elements are located at the targets' range-azimuth

Joint Range-Azimuth Detection

- Frequency diversity: channels are azimuth-dependent and range dependent
- Range-azimuth coupling resolved by using random array and joint range-azimuth detection
- By processing all channels together: achieve range resolution according to total bandwidth



Recovery algorithm via OMP Matrix Approach



$$\min \|\mathbf{X}\|_0 \quad s.t.\ \mathbf{Y}^m=\mathbf{A}^m\mathbf{X}\left(\mathbf{B}^m\right)^T,\quad 0\le m\le M-1.$$

Algorithm 1 OMP for simultaneous sparse matrix recovery

Input: observation matrices $\mathbf{Y}^m$, measurement matrices $\mathbf{A}^m$, $\mathbf{B}^m$, for all $0\le m\le M-1$

Output: index set Λ containing the locations of the non zero indices of $\mathbf{X}$, estimate for sparse matrix $\hat{\mathbf{X}}$

1: Initialization: residual $\mathbf{R}_0^m=\mathbf{Y}^m$, index set $\Lambda_0=\emptyset$, $t=0$

2: Project residual onto measurement matrices:

$$\mathbf{\Psi}=\mathbf{A}^H\mathbf{R}\bar{\mathbf{B}}$$

where $\mathbf{A}=[\mathbf{A}^{0^T}\ \mathbf{A}^{1^T}\cdots\mathbf{A}^{(M-1)^T}]^T$ and $\mathbf{B}=[\mathbf{B}^{0^T}\ \mathbf{B}^{1^T}\cdots\mathbf{B}^{(M-1)^T}]^T$ and $\mathbf{R}=\mathrm{diag}\left([\mathbf{R}_t^0\cdots\mathbf{R}_t^{M-1}]\right)$ is block diagonal

3: Find the two indices $\lambda_t=[\lambda_t(1)\quad\lambda_t(2)]$ such that

$$[\lambda_t(1)\quad\lambda_t(2)]=\arg\max_{i,j}|\mathbf{\Psi}_{i,j}|$$

4: Augment index set $\Lambda_t=\Lambda_t\bigcup\{\lambda_t\}$

5: Find the new signal estimate

$$\alpha=[\alpha_1\ \ldots\ \alpha_t]^T=(\mathbf{D}_t^T\mathbf{D}_t)^{-1}\mathbf{D}_t^T\mathrm{vec}(\mathbf{Y})$$

6: Compute new residual

$$\mathbf{R}_t^m=\mathbf{Y}^m-\sum_{l=1}^{t}\alpha_l\mathbf{a}^m_{\Lambda_t(l,1)}\left(\mathbf{b}^m_{\Lambda_t(l,2)}\right)^T$$

7: Increment t and return to step 2 if $t\le L$, otherwise stop

8: Estimated support set $\hat{\Lambda}=\Lambda_L$

9: Estimated matrix $\hat{\mathbf{X}}$: $(\Lambda_L(l,1),\Lambda_L(l,2))$-th component of $\hat{\mathbf{X}}$ is given by α_l for $l=1,\cdots,L$ while rest of the elements are zero

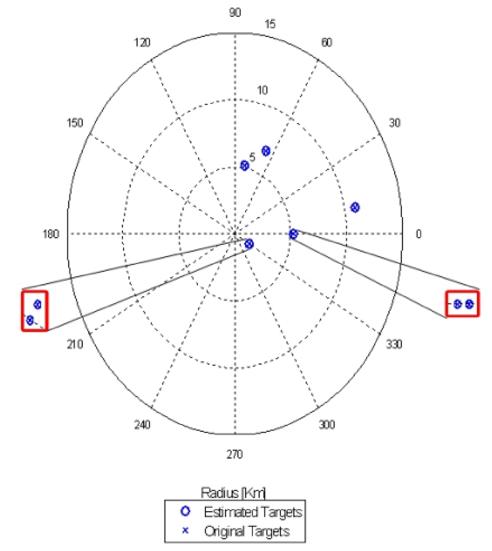
Simulation results

Graph 1: Resolution in range and Azimuth

7 targets including couple of targets with close range and a couple with close azimuth

Spatial Reduction	T=20	M=10	50%
	R=20	Q=10	50%
Time Reduction	N=500	K=250	50%

Total samples reduction: 12.5%

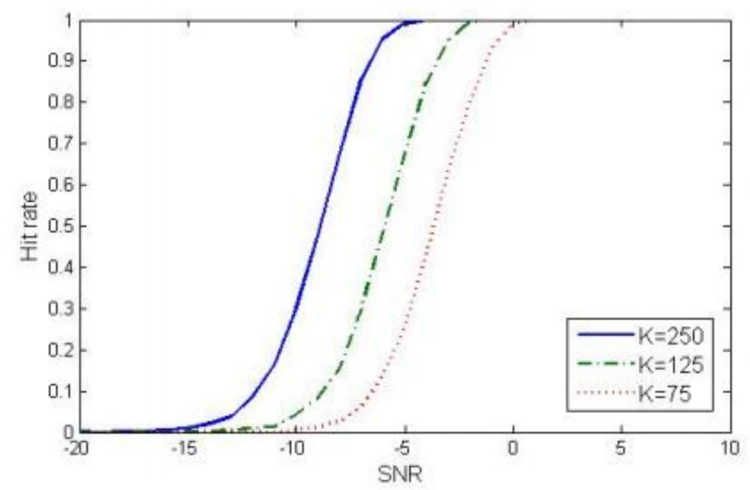


Graph 2: Hit rate vs SNR

10 targets range-azimuth recovery performance

Spatial Reduction	T=20	M=10	50%
	R=20	Q=10	50%
Time Reduction	N=500	K=250	50%
		K=125	25%
		K=75	15%

Total samples reduction: 3.75%



References

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- [3] M. Rossi A.M. Haimovich and Y. C. Eldar, "Spatial compressive sensing for MIMO radar", *Signal Processing IEEE Transactions on*, 2014.