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Carrier Frequency and Bandwidth Estimation of Cyclostationary Multiband Signals

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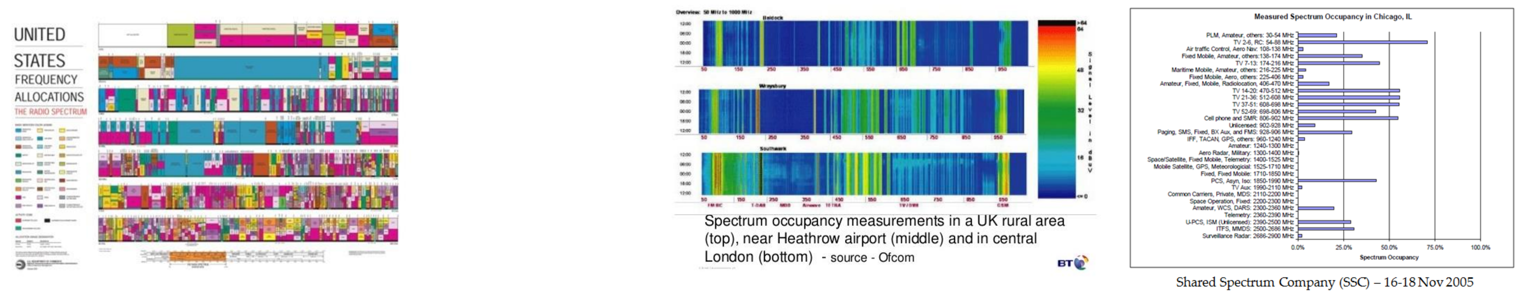
Contributions

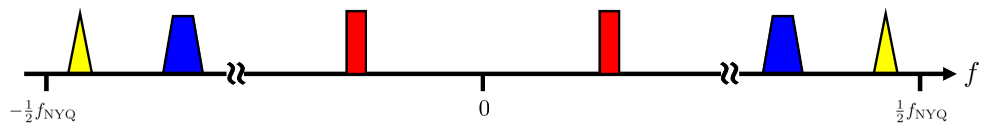
- Simple algorithm for estimating the **number of transmissions**, their **carrier frequencies** and **bandwidths** from the cyclic spectrum of **multiband** signals
- No prior knowledge nor learning requirements
- Applicable to the cyclic spectrum obtained from Nyquist or **sub-Nyquist** samples
- Simulations are performed on both synthetized and **real RF** signals
- **Cyclic spectrum** based estimation outperforms that based on power spectrum

Cognitive Radio and the Multiband Model

- Address the conflict between spectrum saturation and underutilization
- Grant opportunistic access to spectrum "holes" to unlicensed users
- Perform spectrum sensing task efficiently, in real-time and reliably



- Resulting signal sensed by a CR belongs to the multiband model:



Cyclostationarity

- Cyclostationary process: periodic mean and autocorrelation:

$$\mu_x(t+T_0)=\mu_x(t), \qquad R_x(t+T_0,\tau)=R_x(t,\tau)$$

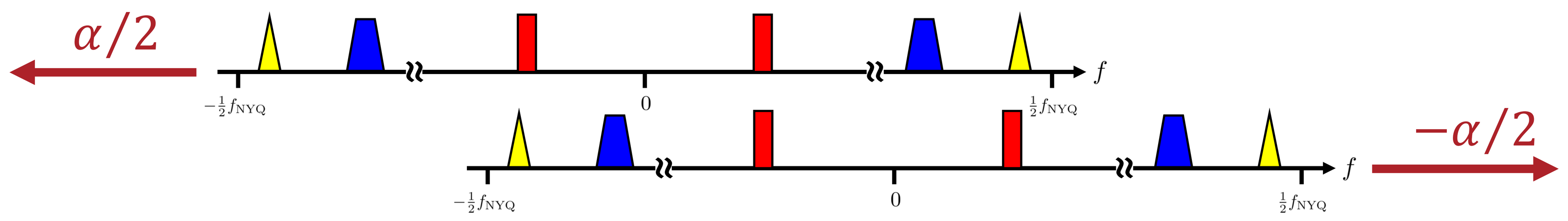
- Cyclic autocorrelations: Fourier series of autocorrelation function

$$R_x(t,\tau)=\sum_\alpha R_x^\alpha(\tau)e^{j2\pi\alpha t}$$

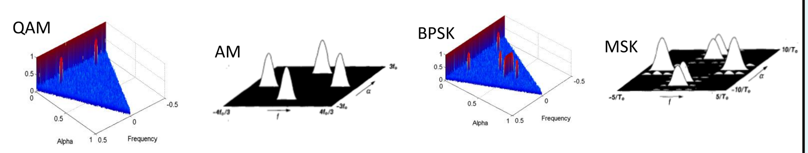
- Cyclic spectrum: Fourier transform of cyclic autocorrelations

$$S_x^\alpha(f)=\int_{-\infty}^{\infty}R_x^\alpha(\tau)e^{-j2\pi f\tau}\mathrm{d}\tau$$

- Alternative definition: cyclic spectrum measures the correlation between two frequency-shifted versions of $x(t)$ as $S_x^\alpha(f)=\mathbb{E}\left[X(f+\frac{\alpha}{2})X^*(f-\frac{\alpha}{2})\right]$



Cyclic spectrum exhibits spectral peaks at frequency locations that depend on carrier frequencies and bandwidths



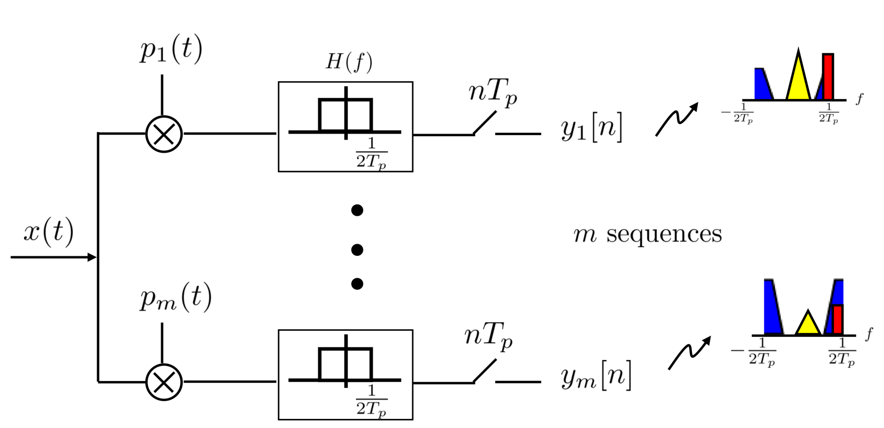
- Cyclic spectrum estimation from Nyquist samples:

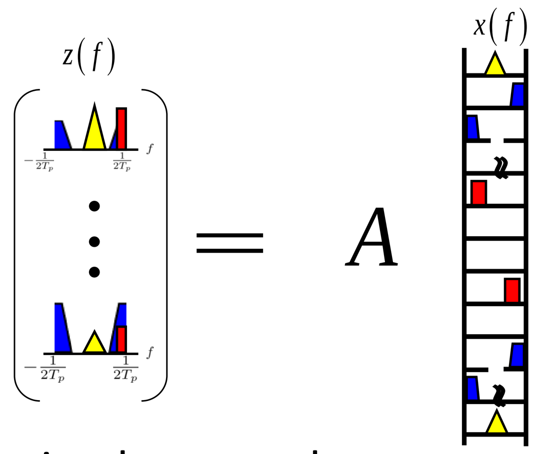
$$\hat{S}_x^\alpha(f)=\sum_{p=1}^{P}\left[X_p(f+\frac{\alpha}{2})X_p^*(f-\frac{\alpha}{2})\right]$$ where $X_p(f)$ is the samples DFT of the pth frame

Goal: estimate number of transmissions, their carrier frequencies and bandwidths from cyclic spectrum

Cyclic Spectrum Recovery from MWC Sub-Nyquist Samples

- MWC (Modulated Wideband Converter) [Mishali '10] : analog front-end: aliases the spectrum so that each band appears in baseband



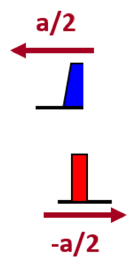


Relation between known samples and unknown signal

- Compute correlations of frequency-shifted versions of the MWC samples:

$$\mathbf{R_z}(a,f)=\mathbf{AR_x}(a,f)\mathbf{A}^H$$

where $\mathbf{R_x}(a,f)=\mathbb{E}\left[\mathbf{x}(f)\mathbf{x}^H(f+a)\right]$

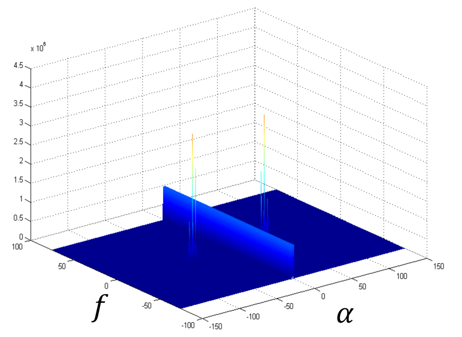


- Reconstruct cyclic spectrum using compressed sensing techniques

Parameter Estimation Algorithm

Step 1: Preprocessing

- Remove DC component of cyclic spectrum ($\alpha=0$)
- Motivation: threshold used for peak detection is proportional to mean signal energy

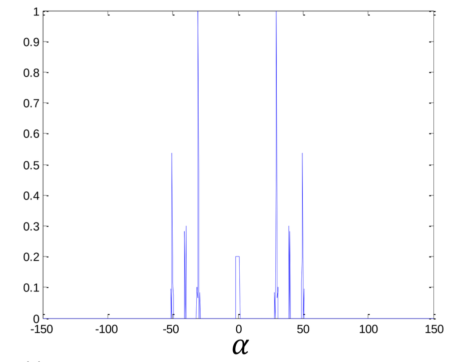


Step 2: Thresholding

- Threshold resulting cyclic spectrum

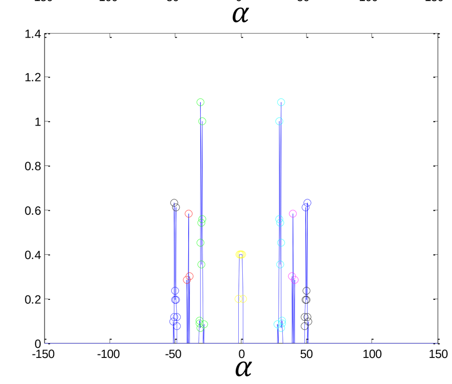
Threshold: design parameter w.r.t. average sample energy

- For each α, retain one peak along frequency axis



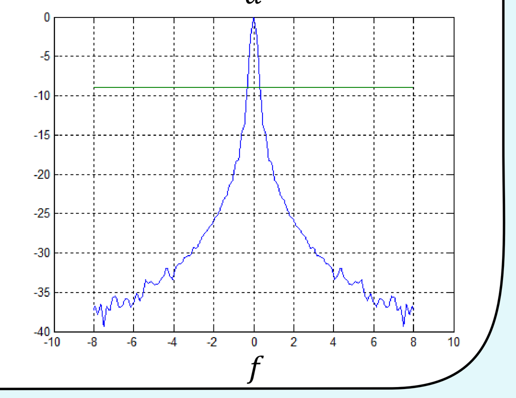
Step 3: Clustering

- Motivation: a cyclic feature yields a cluster of peaks due to finite sensing time
- Clusters number estimation: elbow method
- Clusters separation: k-means
- Remove DC cluster



Step 4: Estimation

- Number of transmissions = half number of clusters
- Carrier: for each cluster, highest peak at $\alpha=2f_c$
- Bandwidth: locating edges of cyclic spectrum at $\alpha=2f_c$



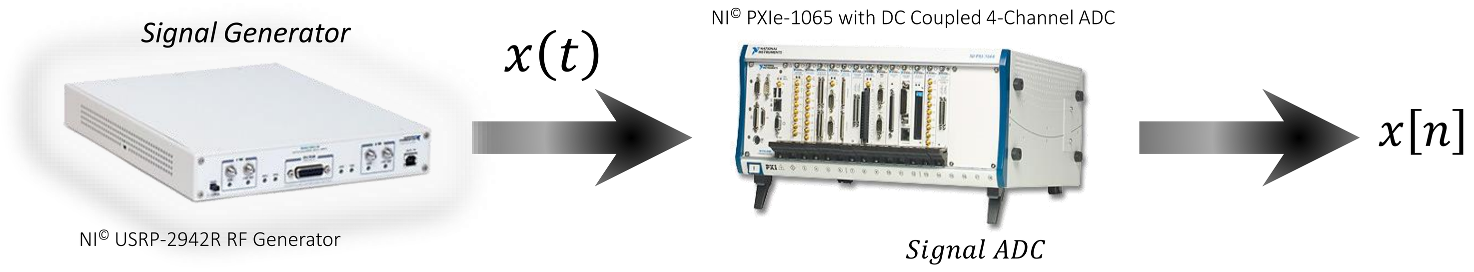
Simulation Results – Nyquist Rate

- Simulated signal (Matlab)

Original carriers	20 MHz	30 MHz	40 MHz
Estimated carriers	20.08 MHz	30.04 MHz	39.94 MHz
Original bandwidths	0.67 MHz	0.67 MHz	0.67 MHz
Estimated bandwidths	0.67 MHz	0.62 MHz	0.71 MHz

Nyquist rate	150 MHz
Sampling rate	150 MHz
Transmissions	3 BPSK
Frames	200
Samples per frame	1000
SNR	-10 dB

- RF signal

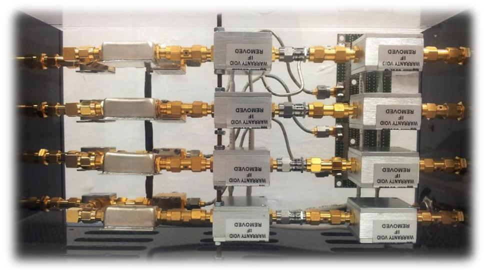


Original carriers	1619.06 MHz	1714.30 MHz	2285.73 MHz
Estimated carriers	1619.46 MHz	1714.73 MHz	2286.30 MHz
Original bandwidths	10 MHz	10 MHz	10 MHz
Estimated bandwidths	9.1 MHz	9.0 MHz	9.7 MHz

Nyquist rate	6.4 GHz
Sampling rate	6.4 GHz
Transmissions	3 BPSK
Frames	100
Samples per frame	7437

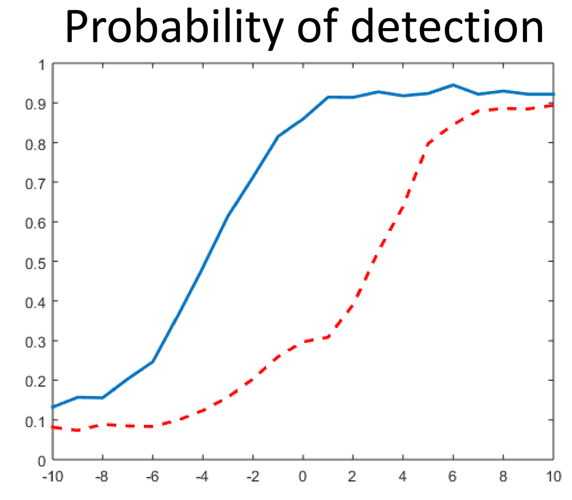
Simulations results – Sub-Nyquist Rate

- Parameter estimation performed on cyclic spectrum recovered from MWC sub-Nyquist samples

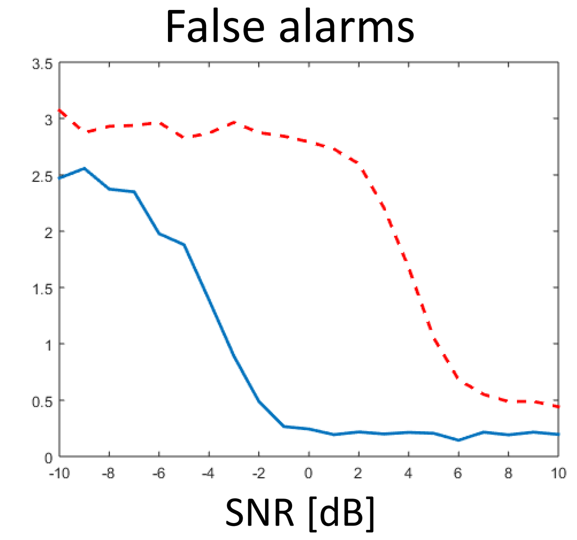


The MWC Card

Probability of detection



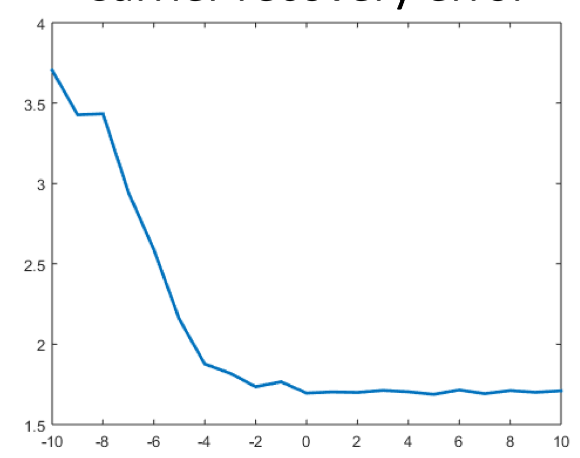
False alarms



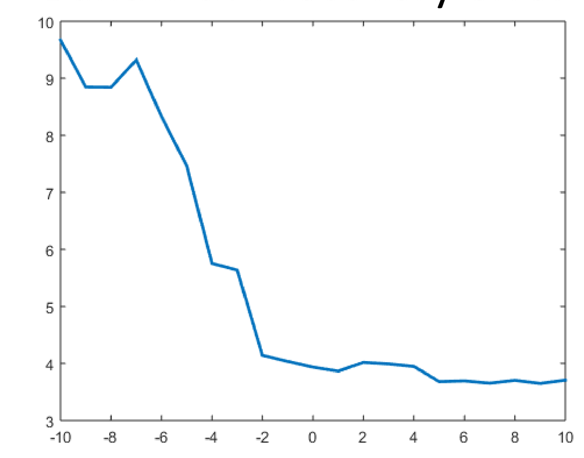
Nyquist rate	1 GHz
Sampling rate	210 MHz
Transmissions	3 BPSK
Frames	100
Samples per frame	121
SNR	-5 dB

21% Nyquist rate

Carrier recovery error



Bandwidth recovery error



References

- [1] W. Gardner, "Statistical spectral analysis: a non probabilistic theory," Prentice Hall, 1988
- [2] M. Mishali and Y. C. Eldar, "From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals," IEEE Journal of Selected Topics in Signal Processing, 2010
- [3] D. Cohen and Y. C. Eldar, "Cyclic spectrum reconstruction from Sub-Nyquist Samples ," IEEE GLOBECOM, 2014