S_X_MPL
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High Spatial Resolution Radar using Thinned Arrays

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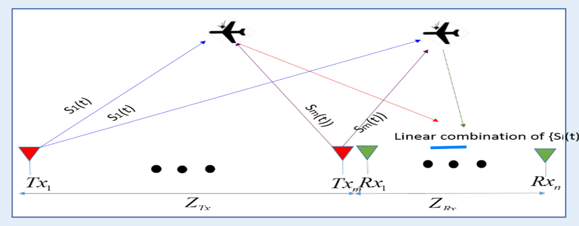
Main Contributions

- Spatial Compressive Sensing framework is employed for a high angular resolution radar that uses fewer elements than a uniform linear array (ULA)
- Previous work on thinned phased arrays lacks efficient direction-of-arrival (DoA) recovery algorithms
- We use multi-branch matched pursuit (MBMP) to recover DoA for a thinned phased array and thinned phased-MIMO hybrid array
- Theoretical performance guarantees to suggest minimum number of array elements for perfect DoA recovery
- Numerical experiments show MBMP outperforms classical beamforming and orthogonal matching pursuit

Antenna Arrays

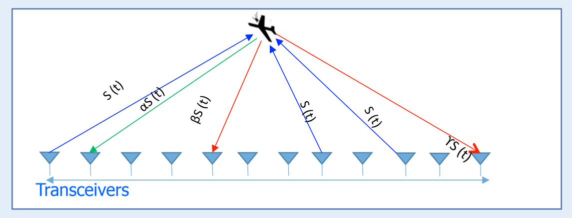
Multiple-Input-Multiple-Output Array

- Uses separate arrays for Tx and Rx
- Each Tx sends a mutually orthogonal waveform
- High parameter identifiability, good for searching large volumes



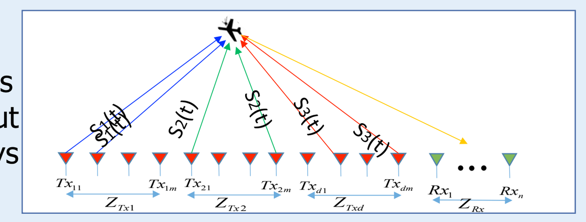
Phased Array (PA)

- Each transmitter sends out same waveform
- High coherent gain, good for tracking targets by forming a focused beam



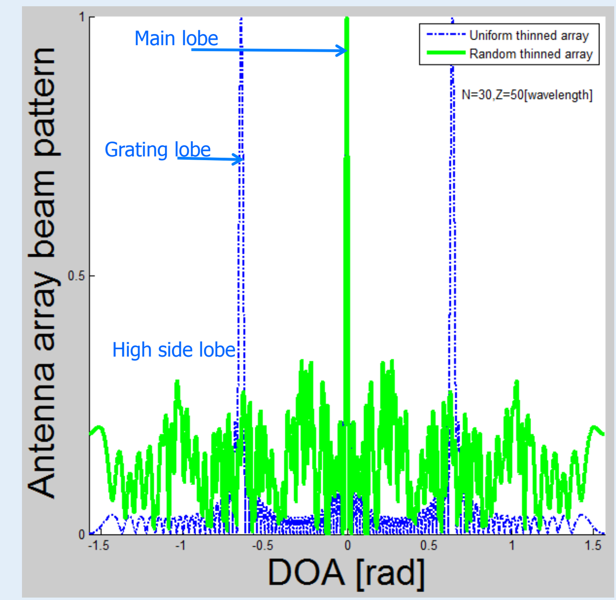
Phased-MIMO Hybrid (PMH)

- Tx array is divided into several subarrays
- Each subarray acts as a PA but sends out waveforms orthogonal to other subarrays
- Combines advantages of MIMO and PA
- Robust to interference

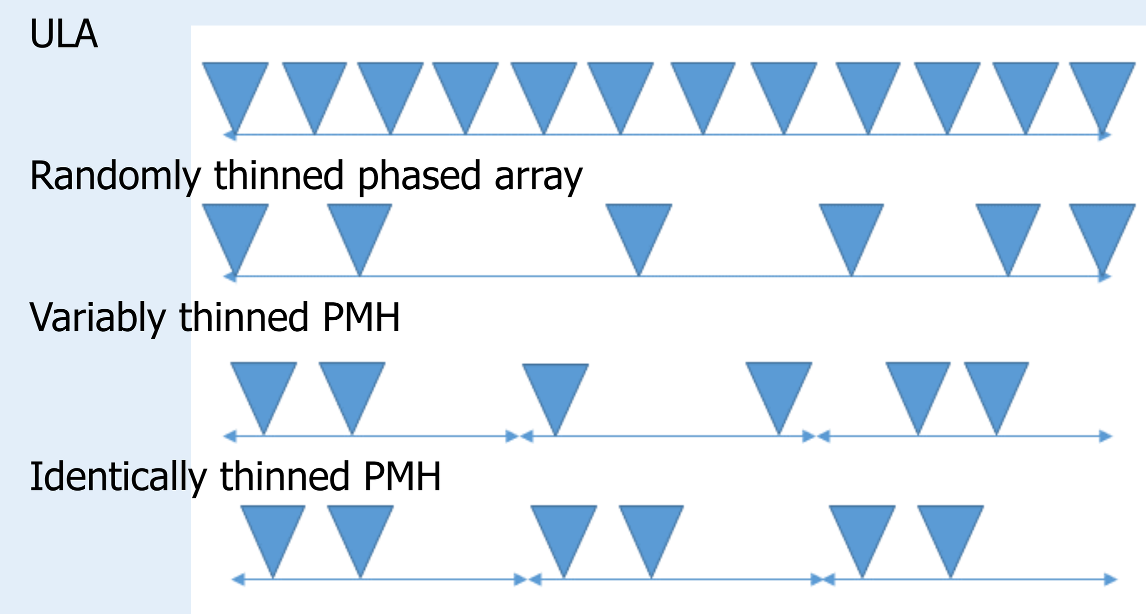


Previous Work

- Uniform thinning of ULA produces grating lobes (multiple beams)
- Random thinning raises sidelobe levels
- Other thinning methods are computationally expensive, control only nearer sidelobes or applicable to small arrays
- Prior DoA recovery algorithms (Beamforming, ESPRIT, MUSIC or OMP) for random arrays are inefficient or inaccurate



Thinned Transmit Array Configurations



Signal Model – Phased Array

- Received signal vector for P pulses and K targets:

$$\boldsymbol{r}(t) = \sum_{p=0}^{P-1}\sum_{k=1}^{K} x_{k,p}\boldsymbol{b}(\theta_k)(\boldsymbol{c}^T(\theta_k)\boldsymbol{w})\boldsymbol{s}(t-pT) + \boldsymbol{e}(t)$$

(Target reflectivity: $x_{k,p}$; Receive steering vector: $\boldsymbol{b}$; Transmit steering vector: $\boldsymbol{c}$; Beamforming weights: $\boldsymbol{w}$; Transmit signal: $\boldsymbol{s}$)

- Received pth signal after matched filtering:

$$\boldsymbol{y}_p = vec\left[\sum_{p=0}^{P-1}\sum_{k=1}^{K} x_{k,p}g(\theta_k)\boldsymbol{b}(\theta_k) + \int \boldsymbol{e}(t)\boldsymbol{s}^H(t-pT)dt\right]$$

$g(\theta_k) = c^t(\theta_k)w$

- Received signal matrix:

$$\boldsymbol{Y} = \tilde{\boldsymbol{A}}(\theta)\tilde{\boldsymbol{X}} + \boldsymbol{E}, where\ \boldsymbol{A}(\theta) = [\boldsymbol{a}(\theta_1),\dots,\boldsymbol{a}(\theta_K)],$$
$$\boldsymbol{a}(\theta) = g(\theta)\boldsymbol{b}(\theta), \boldsymbol{E} = [\boldsymbol{e}_1,\dots,\boldsymbol{e}_P], \boldsymbol{e}_p = vec[\textstyle\int \boldsymbol{e}(t)\boldsymbol{s}^H(t-pT)dt]$$

Signal Model – PMH

- Transmit signal of the kth subarray:

$$\boldsymbol{s}_k(t) = \sqrt{\frac{M}{D}}\boldsymbol{s}_k(t)\boldsymbol{w}_k^T$$

(Number of transmitters: M; Number of subarrays: D)

- Transmit steering vector:

$$\boldsymbol{c}(\theta) = [\boldsymbol{w}_1^H\boldsymbol{h}_1(\theta),\dots,\boldsymbol{w}_D^H\boldsymbol{h}_D(\theta)]^T$$

(Transmit steering vector of subarry: $\boldsymbol{h}_1$)

- Received pth signal after matched filtering:

$$\boldsymbol{r}(t) = \sum_{p=0}^{P-1}\sum_{k=1}^{K}\sqrt{\frac{M}{D}}x_{k,p}\boldsymbol{b}(\theta_k)(\boldsymbol{c}(\theta_k)\odot\boldsymbol{f})^T\boldsymbol{s}(t-pT) + \boldsymbol{e}(t)$$

(Waveform diversity vector: $\boldsymbol{f}$)

- Received signal matrix:

$$\boldsymbol{Y} = \tilde{\boldsymbol{A}}(\theta)\tilde{\boldsymbol{X}} + \boldsymbol{E}, where:\ \boldsymbol{A}(\theta) = [\boldsymbol{a}(\theta_1),\dots,\boldsymbol{a}(\theta_K)],$$
$$\boldsymbol{a}(\psi_g) = (\boldsymbol{c}(\boldsymbol{\psi}_g)\odot\boldsymbol{f}(\boldsymbol{\psi}_g))\otimes\boldsymbol{b}(\theta), f(\theta) = [e^{-\Delta_1(\theta)},\dots,e^{-j\Delta_D(\theta)}]^T$$

Spatial Compressive Sensing

- Assume the target locations lie on a dense grid of G points
- Measurements in the discretization framework (**X** is sparse):

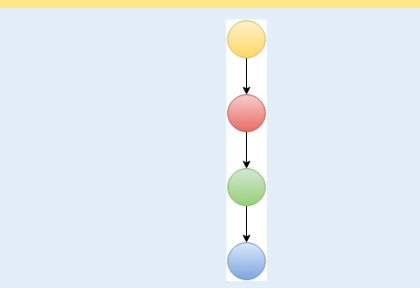
$$Y = AX + E$$

- Nyquist arrays: **A** is a square matrix and G=N
  - Beamforming can retrieve the support of **X**
- When A is a fat matrix, SCS framework solves optimization problem

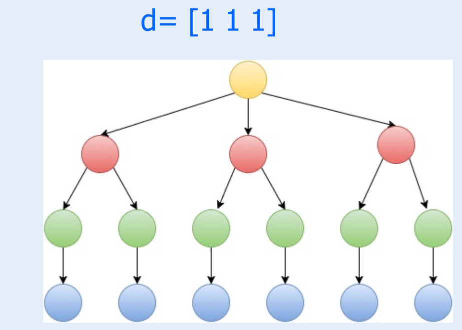
$$\underset{\mathbf{x}}{\text{minimize}}\ \|\mathbf{x}\|_1 \quad \text{subject to}\ \|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2 \le \epsilon.$$

MBMP Algorithm- Visualization

- The branching structure is defined by the branching vector d.
- The vector determines the number of branches in each iteration.
- In each iteration, the MBMP algorithm choose number of peaks (maximum value of the inner product) determined by the branching vector d.
- Highly efficient than other greedy algorithms and well-suited for tree-based search.



d= [1 1 1]



d= [3 2 1]

Theoretical Performance Guarantees

- We use probabilistic theory of antenna arrays (Lo, 1964) to guarantee SCS performance for PA

Theorem *Let the locations $\{\zeta\}_{i=1}^N$ of the transmit and receive elements of a phased array be drawn i.i.d. from a distribution $p(\zeta)$. Let $\hat{\mathbf{x}}$ be the solution of (15). Then, with probability at least $1-\varepsilon$, we have*

$$\|\mathbf{x}-\hat{\mathbf{x}}\|_2 \le C_0\varepsilon/C_N(\mathbf{w}),$$

as long as the number of elements N satisfies

$$N \ge C(K-0.5)^2\ln\left(\frac{G}{\varepsilon}\right)^4,$$

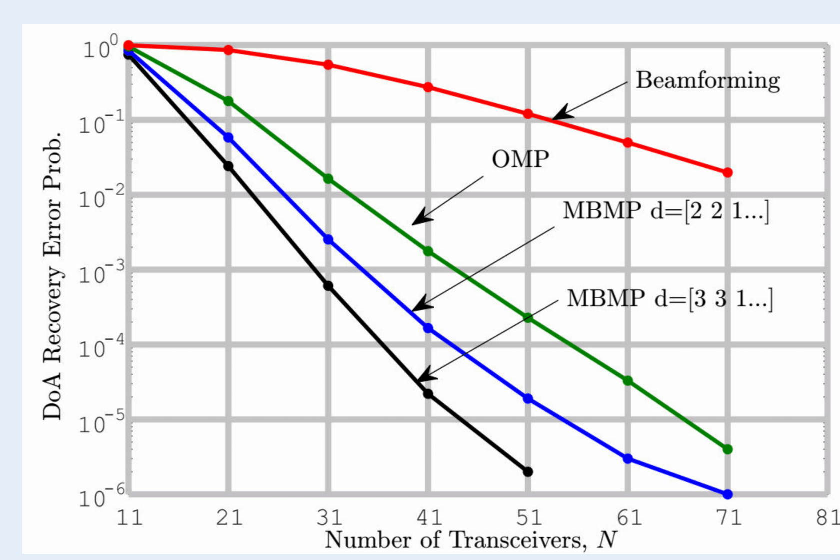
where $C = (43+12\sqrt{7})/16$, C_0 is a positive constant that depends only on ε, $C_N(\mathbf{w})$ is the directivity or the gain that depends on the choice of $\mathbf{w}$, G is the grid size, and K is the sparsity of the vector $\mathbf{x}$.

Comparison of MIMO and PA performance guarantees

Type of arrays	*Uniform*	*Non-uniform*
Thinned MIMO	$MN \ge C\left(K-\frac{1}{2}\right)^2\left[\ln\frac{\sqrt{\pi}G}{2\epsilon}+\frac{1}{2}\ln\left(2\ln\frac{\sqrt{\pi}G}{2\epsilon}\right)\right]$	$MN \ge CK\log^2\left(\frac{cG}{\epsilon}\right)$
Thinned PA	$N \ge C(K-\frac{1}{2})^2\left[4\ln\left(\frac{G}{\epsilon}\right)\right]$	$N \ge C_1(K(\ln(G/\epsilon)+\ln(K)\ln(K/\epsilon)))$

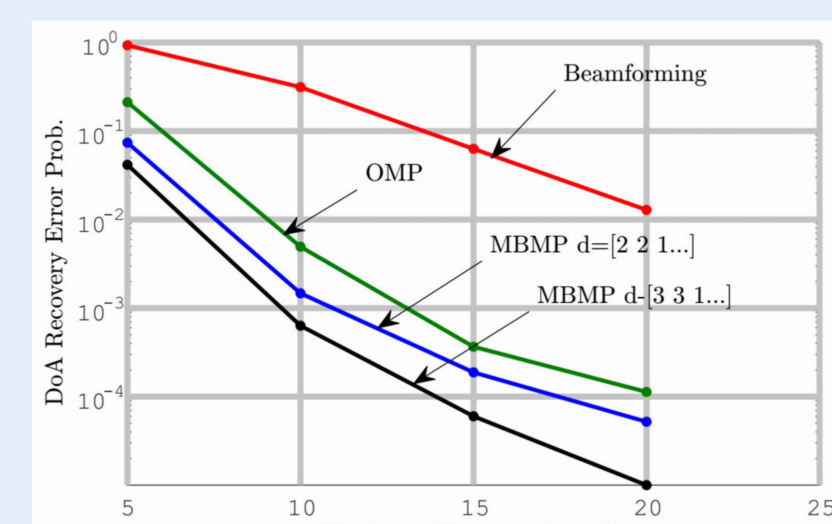
NE1 - Phased Array

- DoA recovery error vs. number of transceivers
- Absolute recovery via ULA corresponds to 81 transceivers.



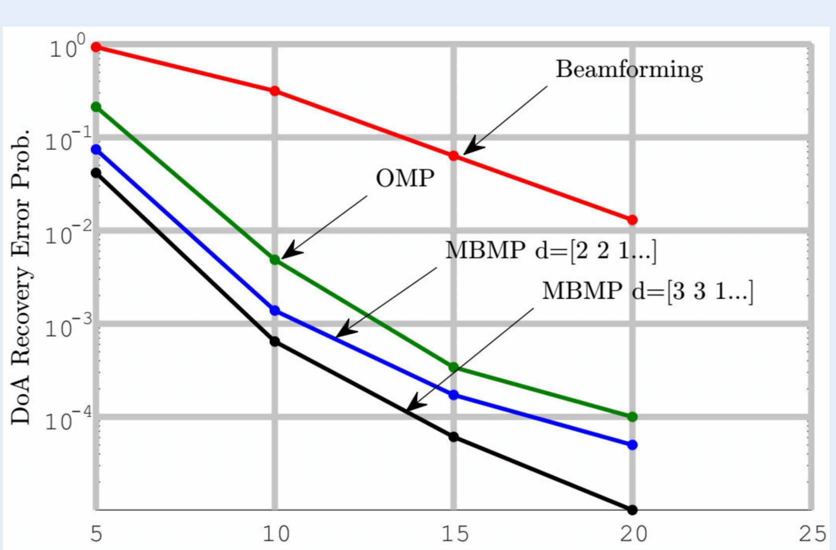
NE2 – Variably Thinned Hybrid

- DoA recovery error vs. number of transmitters, divided into 5 subarrays.
- Absolute recovery is achieved using a ULA with 25 transmitters.
- CDM signals were used here.



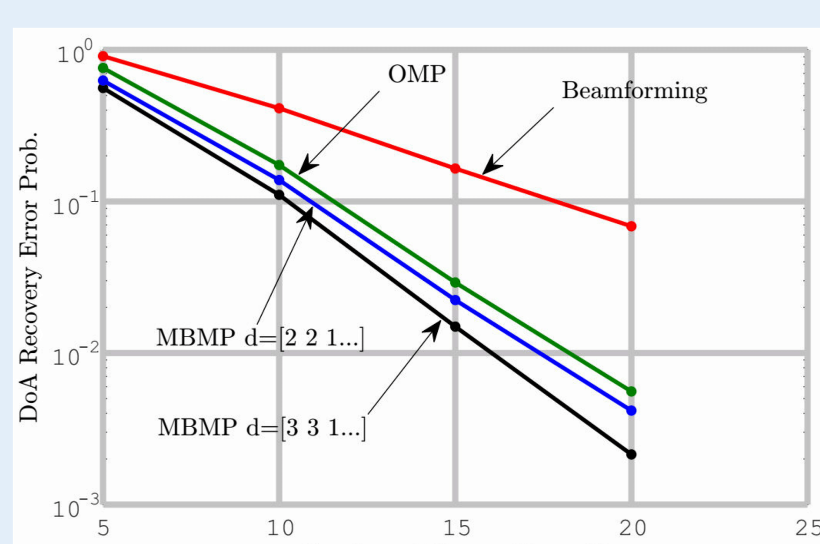
NE3 – Variably Thinned Hybrid

- DoA recovery error vs. number of transmitters, divided into 5 subarrays.
- Absolute recovery is achieved using a ULA consists of 25 transmitters.
- FDM signals were used.



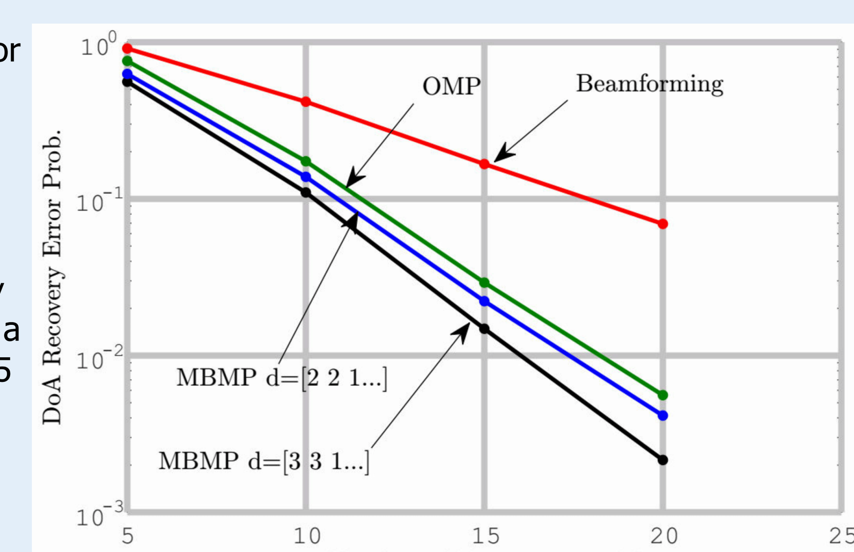
NE4 – Identically Thinned Hybrid

- DoA recovery error vs. number of transmitters, divided into 5 subarrays.
- Absolute recovery is achieved using a ULA consists of 25 transmitters.
- CDM signals were used.



NE5 – Uniform Thinned Hybrid

- DoA recovery error vs. number of transmitters, divided into 5 subarrays.
- Absolute recovery is achieved using a ULA consists of 25 transmitters.
- FDM signals were used.



Summary

- Examined various array architectures for DoA recovery with few elements.
- Theoretical performance guarantees derived for thinned PA.
- MBMP outperforms classic beamforming, MUSIC and OMP algorithms.
- Further investigation into theoretical performance and other array architectures required.