Channel Cycle Time: A New Measure of Short-Term Fairness

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***Abstract*—This paper puts forth a new metric, dubbed channel cycle time (CCT), to measure the short-term fairness of communication networks. CCT characterizes the average duration between two consecutive successful transmissions of a user, during which all other users successfully accessed the channel at least once. In contrast to existing short-term fairness measures, CCT provides more comprehensive insight into the transient dynamics of communication networks, with a particular focus on users' delays and jitter. To validate the efficacy of our approach, we analytically characterize the CCTs for two classical communication protocols: slotted Aloha and CSMA/CA. The analysis demonstrates that CSMA/CA exhibits superior short-term fairness over slotted Aloha. Beyond its role as a measurement metric, CCT has broader implications as a guiding principle for the design of future communication networks by emphasizing factors like fairness, delay, and jitter in short-term behaviors.**

***Index Terms*—short-term fairness, channel cycle time, multiple access protocols.**

I. INTRODUCTION

Fairness refers to the principle of providing equal access to resources without discrimination [1]–[3]. In multiple-access networks, where multiple users share the scarce radio frequency resource and simultaneous transmissions result in collisions, fairness is a critical design principle that guarantees the timely and reliable communication of each user, without suffering undue delay or interference from other users [4]–[8].

Fairness in network resource allocation can be classified as long-term or short-term, depending on the time scale over which channel resources are allocated [9]–[12].

- Long-term fairness is concerned with allocating channel resources fairly over a sufficiently long period of time. It characterizes the ergodic behavior of a multiple-access control (MAC) protocol and is the predominant focus of the current literature.
- Short-term fairness, in contrast, is concerned with ensuring the fair allocation of network resources in the immediate or near term. It characterizes the transient behavior of a MAC protocol.

Achieving short-term fairness is more challenging. A short-term fair MAC protocol is also long-term fair, but the reverse is not always true. A user may monopolize the channel in short periods to the detriment of other users, even if it has the same channel occupancy rate as other users over a long period.

The proliferation of machine-type and low-latency applications has catalyzed a notable shift in the evaluation of MAC protocols, with an increasing emphasis on short-term fairness over long-term fairness. This shift recognizes the need for MAC protocols to prioritize near-term performance and responsiveness in future networks, where delays and latency can have significant impact on user experience.

To measure short-term fairness, [5], [13] proposed a fairness index: the number of inter-transmissions that other hosts may perform between two consecutive transmissions of a given host. Consider a multiple-access network with three users A, B, and C, and a pattern of successful transmissions 'ABCABBCBAC'. The number of inter-transmissions for users A, B, and C is $\{2, 4\}$, $\{2, 0, 1\}$, and $\{3, 2\}$, respectively. The authors proposed to characterize the short-term fairness by the probability density function (PDF) of the number of inter-transmissions of all users. This approach has two main limitations. First, it considers only successful transmissions, ignoring the time consumed by these transmissions. As a consequence, a poorly designed MAC protocol with frequent collisions could have the same short-term fairness as a well-designed MAC protocol, if measured by the number of inter-transmissions. Second, obtaining the entire PDF of the number of inter-transmissions can be cumbersome, especially when comparing the short-term fairness of two MAC protocols. In [5], the authors proposed using the average number of inter-transmissions as a simpler way to evaluate short-term fairness, with a smaller value indicating better fairness. However, the average number of inter-transmissions loses too much information compared to the entire PDF (see the example in Section II-B), hence is inadequate to fully characterize the short-term fairness of a MAC protocol.

The authors in [4] proposed a "renewal reward method" to

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measure short-term fairness, where the transitions of channel occupation are assigned rewards. In particular, the reward a user receives from a successful transmission depends on the number of transmissions of other users from the user's last successful transmission. The reward function is designed to be non-decreasing and reaches the minimum when the same user keeps the channel. A MAC protocol that yields higher rewards is considered short-term fairer. However, similar to the number of inter-transmissions, the renewal reward method considers only successful transmissions. It further requires the reward function to be carefully designed for individual MAC protocols to be evaluated.

Another, and perhaps the simplest approach to measure short-term fairness is to reuse the long-term fairness measures in a short period of time. Specifically, for each epoch, we count the number of successful transmissions of each user over a short period and compute the α fairness [6], [14], Jain's index [15], or any other traditional fairness measure [1], [16]–[20]to capture the short-term behaviors of the network. Compared with aforementioned approaches that consider only successful transmissions, this method captures the transient behavior of the network by setting a time window to count the number of successful transmissions. The challenge, however, lies in determining an appropriate duration of the time window. At different epochs, the network state varies, hence the amount of time required to measure the transient behavior of the network is different. For a given MAC protocol, determining the appropriate time window to evaluate short-term fairness can be a non-trivial problem in itself.

In this paper, we put forth a new metric, dubbed channel cycle time (CCT), to measure short-term fairness. CCT is a time measure that characterizes the average duration between two successful transmissions of a user, during which all other users have successfully accessed the channel at least once. Compared with existing short-term fairness measures, our measure has three salient features.

- CCT is a single real value that is easy to compute. This facilitates easy comparison of the short-term performance of different MAC protocols.
- CCT effectively captures the transient behavior of a MAC protocol under varying network states, reflecting the average time it takes for all users to successfully transmit at least once.
- CCT provides a comprehensive picture of the short-term fairness of a MAC protocol, with an emphasis on users' delay and jitter.

To demonstrate the effectiveness of our new approach, we consider two homogeneous multiple-access networks operated with two classical MAC protocols: slotted Aloha and carrier-sense multiple access with collision avoidance (CSMA/CA), respectively. The closed-form CCT is derived for both cases. It is shown that CSMA/CA is a short-term fairer protocol than slotted Aloha.

Beyond its role as a short-term fairness measurement, CCT can provide guidelines for MAC protocol development. For both homogeneous networks employing slotted Aloha and CSMA/CA, we optimize channel access parameters, such as transmission probability and contention window size, using the CCT, yielding their revamped versions that place a premium on short-term fairness. Overall, channel cycle time holds significant potential as a novel evaluation and design principle for future communication networks, prioritizing the enhancement of short-term characteristics such as fairness, delay, and jitter.

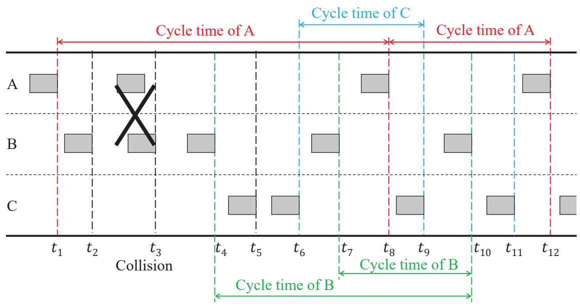

Fig. 1: Illustrations of the refresh moment and cycle time of users in a multiple-access network ($N = 3$).

II. Channel Cycle Time

A. Definition of CCT

This section formally introduces the concept of channel cycle time. We consider a multiple-access network, wherein N users communicate with a common access point (AP) in a shared channel using a given MAC protocol. To start with, we define the "refresh moment", "refresh time", and "cycle time" for each user.

Definition 1 (Refresh moment)**.** *A moment is a refresh moment of a user if and only if*

- *A successful channel access of the user ends at this moment.*
- *The next successful channel access belongs to other users.*

Definition 2 (Refresh time)**.** *A refresh time of the n-th user, denoted by $\mathcal{T}_n$, is defined as the time between two consecutive refresh moments of the n-th user.*

An example is given in Fig. 1, where there are three users A, B, and C in the network. As can be seen, the pattern of successful transmissions is 'ABBCCBACBCA'. Based on Definitions 1 and 2, the refresh moments of users A, B, and C are $\{t_1, t_8, t_{12}\}$, $\{t_4, t_7, t_{10}\}$, and $\{t_6, t_9, t_{11}\}$, respectively, and the corresponding refresh times are $\{t_8 - t_1, t_{12} - t_8\}$, $\{t_7 - t_4, t_{10} - t_7\}$, and $\{t_9 - t_6, t_{11} - t_9\}$, respectively.

Consider any two epochs t and t', we denote by $\{M_{n'}(t, t') : n' = 1, 2, ..., N\}$ the number of successful channel accesses of all users between t and t'. The cycle time of a user can be defined as follows.

Definition 3 (Cycle time)**.** *Consider two refresh moments t_0 and t_1, $t_0 < t_1$, of the n-th user. The cycle time of the n-th user, denoted by Γ_n, is a random variable.*

- *When t_0 and t_1 are consecutive refresh moments, $|t_0 - t_1|$ is a cycle time of the n-th user if and only if*

$$\min_{n'} M_{n'}(t_0, t_1) > 0. \tag{1}$$

- *When t_0 and t_1 are nonconsecutive, $|t_0 - t_1|$ is a cycle time of the n-th user if and only if*

$$\begin{cases} \min_{n'} M_{n'}(t_0, t_1) > 0, \\ \min_{n'} M_{n'}(t_0, t') = 0, \end{cases} \tag{2}$$

where t' : $t_0 < t' < t_1$ is the closest refresh moment of the n-th user to t_1.

Succinctly speaking, Γ_n measures the duration between two closest refresh moments of the n-th user, in which all other users successfully access the channel at least once. Definition 3 also indicates that the cycle time of a user consists of several refresh time. That is, $\Gamma_n = \sum_{i=1}^{I_r} \mathcal{T}_{n,i}$, where I_r is the number of refresh time that Γ_n contains. Therefore, the average cycle time can be calculated as

$$\mathbb{E}[\Gamma_n] = \mathbb{E}_{I_r, \mathcal{T}_{n,i}} \left[\sum_{i=1}^{I_r} \mathcal{T}_{n,i} \right]. \quad (3)$$

Definition 4 (Channel cycle time)**.** *In a multiple-access network with N users, suppose the users go through $\xi_n(T)$, $n = 1, 2, ..., N$, cycle times within a period of time T. The channel cycle time (CCT) of the network is defined as the average cycle time of all users:*

$$\Psi = \lim_{T \to \infty} \frac{\sum_{n=1}^{N} \xi_n(T) \mathbb{E}[\Gamma_n]}{\sum_{n=1}^{N} \xi_n(T)}. \quad (4)$$

For any MAC protocol, a smaller CCT indicates better short-term fairness. In particular, when the packet duration of each user is fixed, round-robin TDMA is the short-term fairest protocol. In this case, CCT is lower bounded by $\Psi \geq \sum_{n=1}^{N} \ell_n$, where ℓ_n denotes the duration of the data packet of the n-th user.

B. Superiority of CCT over existing measures

Consider a network with two users A and B, the packet duration of which is ℓ_A and ℓ_B, respectively. Suppose there are two TDMA protocols: the first protocol operates as 'AABBAABB...', while the second operates as 'ABABABAB...' Which protocol is fairer?

It is easy to see that the two protocols are equally fair in the long term. To measure the short-term fairness,

- If we use the average number of inter-transmissions, both protocols have a measure of 1, meaning that they are equally fair.
- In contrast, the CCT of the two protocols are $2(\ell_A + \ell_B)$ and $(\ell_A + \ell_B)$, respectively, indicating that the second protocol is short-term fairer, which is more in line with our intuition.
- Finally, if we use traditional long-term fairness measures with a time window, the choice of window size has a significant impact on the results obtained, as stated in the introduction. Specifically, if we choose the window size to be $2(\ell_A + \ell_B)$, the two protocols are equally fair. Varying the window size yields different results.

In the following two sections, we delve into homogeneous networks operated with two classical MAC protocols: slotted Aloha and CSMA/CA. Our focus will be on analyzing their short-term fairness via CCT. To extract both analytical findings and deeper insights, we will consistently approach a saturated traffic scenario, wherein each user's queue is brimming and there is always a packet ready for transmission. Furthermore, we assume that the duration of data packets for each user remains consistent and is denoted by ℓ_{pkt}.

III. CCT of Slotted Aloha

Slotted Aloha is a simple and efficient MAC protocol. When operated with slotted Aloha, time is divided into equal-sized slots with duration T_{slot}, and users can transmit only at the beginning of a time slot. Consider a network with N users and let $T_{\text{slot}} = \ell_{\text{pkt}}$. At the beginning of any time slot, each user transmit a packet with probability p. A packet can be successfully transmitted only when all other users are silent in this slot (the probability of which is $Np(1-p)^{N-1}$).

A. CCT of slotted Aloha

In slotted Aloha, the transmissions across different slots are independent. Thus, in a cycle time, $\mathcal{T}_{n,i}, i = 1, 2, \cdots, I_r$ are independent and identically distributed (i.i.d.) random variables. Eq. (3) can be refined as

$$\mathbb{E}[\Gamma_n] = \mathbb{E}[I_r] \cdot \mathbb{E}[\mathcal{T}_n]. \quad (5)$$

The average cycle time of a user can be obtained by deriving $\mathbb{E}[I_r]$ and $\mathbb{E}[\mathcal{T}_n]$, respectively.

Lemma 1. *In slotted Aloha, the average time it takes to successfully transmit a packet is*

$$\overline{T} = \frac{T_{\mathit{slot}}}{Np(1-p)^{N-1}}. \quad (6)$$

Proof. Assume there are K slots between two consecutive successful transmissions. The distribution of K is given by

$$\Pr(K=k) = [1 - Np(1-p)^{N-1}]^{k-1} \cdot Np(1-p)^{N-1}, k = 1, 2, \cdots$$

Thus, we have $\overline{T} = \mathbb{E}(K) \cdot T_{\text{slot}}$, which gives us (6). ■

Proposition 2. *When operated with slotted Aloha, the average refresh time of each user is given by*

$$\mathbb{E}[\mathcal{T}] = \frac{N}{(N-1)p(1-p)^{N-1}} \cdot T_{\mathit{slot}}. \quad (7)$$

Proof. Without loss of generality, we consider two adjacent refresh moments of a user A. When there is a successful transmission, we denote the probabilities that the packet is from A and other users by $p_{A|S}$ and $p_{\overline{A}|S}$, respectively. An immediate result is that $p_{A|S} = 1/N$ and $p_{\overline{A}|S} = (N-1)/N$. Suppose the refresh time of user A contains n_A ($n_{\overline{A}}$) consecutive packets successfully transmitted by A (other users), where $n_A, n_{\overline{A}} \in \{1, 2, 3, \cdots\}$.

The refresh time of user A can then be written as

$$\mathcal{T}_A = \sum_{i=1}^{n_A} T_{A,i} + \sum_{j=1}^{n_{\overline{A}}} T_{\overline{A},j},$$

where $T_{A,i}$ ($T_{\overline{A},j}$) is the time it takes for user A (other users) to transmit the i-th (j-th) successful packet. We emphasize that $T_{A,i}$ and $T_{\overline{A},j}$ may not equal T_{slot} because of collisions.

Then, $\mathbb{E}[\mathcal{T}_A]$ is given by

$$\begin{aligned} \mathbb{E}[\mathcal{T}_A] &= \sum_{n_A=1}^{\infty} \sum_{n_{\overline{A}}=1}^{\infty} (n_A \overline{T}_A + n_{\overline{A}} \overline{T}_{\overline{A}}) \cdot \left(\frac{1}{N}\right)^{n_A} \left(\frac{N-1}{N}\right)^{n_{\overline{A}}} \\ &= \frac{N^2}{N-1} \cdot \frac{\overline{T}_A + (N-1)\overline{T}_{\overline{A}}}{N}, \end{aligned} \quad (8)$$

where $\overline{T}_A$ and $\overline{T}_{\overline{A}}$ denote the average values of $\{T_{A,i}\}$ and $\{T_{\overline{A},j}\}$, respectively (they also represent the average time it takes to successfully transmit a packet of A and other users, respectively). As a result, $\frac{\overline{T}_A+(N-1)\overline{T}_{\overline{A}}}{N}$ is exactly the average time of a successful transmission, that is, $\frac{\overline{T}_A+(N-1)\overline{T}_{\overline{A}}}{N}=\overline{T}$.

It can be shown that $\mathbb{E}[\mathcal{T}_n]$ is the same for all users with slotted Aloha. Substituting (6) into (8) gives us (7). ■

Proposition 3. *The average number of refresh time that one cycle time contains in slotted Aloha is given by*

$$\mathbb{E}[I_r]=\frac{N-1}{N}(1+H_{N-1}), \tag{9}$$

where $H_{N-1}\triangleq\sum_{i=1}^{N-1}\frac{1}{i}$ is the $(N-1)$-th harmonic number.

Proof. To conserve space, the proof is presented in our technical report [21]. ■

Theorem 4. *The CCT of slotted Aloha is*

$$\Psi_{\textit{slotted-Aloha}}=\frac{1+\sum_{i=1}^{N-1}\frac{1}{i}}{p(1-p)^{N-1}}\cdot T_{\textit{slot}}. \tag{10}$$

Proof. The average cycle time of the n-th user $\mathbb{E}[\Gamma_n]$ can be obtained by substituting (7) and (9) into (5). Note that $\mathbb{E}[\Gamma_n]$ is same for all users. As per Definition 4, the CCT of slotted Aloha is exactly $\mathbb{E}[\Gamma_n]$. ■

B. CCT-optimal slotted Aloha

Given the closed-form channel cycle time, we next investigate the CCT-optimal slotted Aloha. That is, we optimize the transmission probability p to obtain the short-term fairest slotted Aloha.

Differentiating (10) with respect to p and setting the results to 0, it is easy to find that the optimal $p^*=\frac{1}{N}$, in which case slotted Aloha has the minimum channel cycle time:

$$\Psi^*_{\text{slotted-Aloha}}=\frac{N(1+H_{N-1})}{(1-\frac{1}{N})^{N-1}}T_{\text{slot}}=\frac{N(1+H_{N-1})}{(1-\frac{1}{N})^{N-1}}\ell_{\text{pkt}}.$$

It is worth noting that the throughput of slotted Aloha reaches the maximum when the average number of transmission trials per slot $G=1$. For the network with N users, G can be calculated as

$$G=\sum_{i=1}^{N}i\cdot\binom{N}{i}p^i(1-p)^{N-i}=Np.$$

Therefore, for slotted Aloha, the transmission probability that achieves the minimum CCT also gives us the maximum throughput.

In contrast, other short-term metrics fail to offer such valuable insights. The fundamental reason for this lies in the fact that CCT also captures the transmission delay and jitter, establishing an intrinsic connection with throughput. Other metrics solely focus on successfully transmitted packets without any regard for time, constraining their capacity to comprehensively characterize the short-term behavior of a MAC protocol.

IV. CCT OF CSMA/CA

CSMA/CA is the MAC protocol widely used in Wi-Fi networks [22]. In CSMA/CA, each terminal carrier senses the channel before transmission and retransmits after a binary exponential backoff in the case of transmission failures. That is, the average backoff time is doubled after a transmission failure to reduce the collision probability.

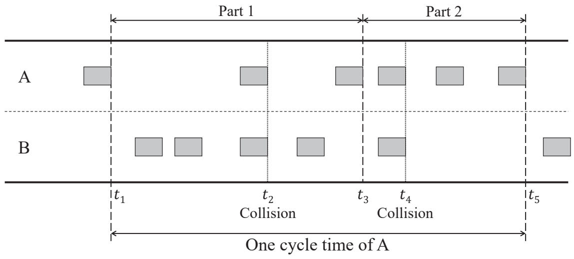

Fig. 2: In CSMA/CA, the cycle time of a user can be divided into two parts.

Compared to slotted Aloha, the analysis and evaluation of CSMA/CA pose more complex challenges. This stems from the fact that successful transmissions in CSMA/CA are considerably interrelated, in contrast to the independent transmissions in slotted Aloha. Consequently, a distinct approach must be adopted to dissect the CCT of CSMA/CA.

To gain analytical insights and derive closed-form expressions, this section will concentrate on a network featuring two users, designated as A and B. For such a two-user network, the concepts of cycle time and refresh time become interchangeable, yielding $\mathbb{E}[\Gamma_n]=\mathbb{E}[\mathcal{T}_n]$. With this property, we will meticulously analyze CSMA/CA in both the RTS/CTS mode and the basic mode (without RTS/CTS). For networks involving more users, we will resort to simulations to evaluate their channel cycle time.

A. CCT of CSMA/CA

We first evaluate CSMA/CA in the RTS/CTS mode. An important observation is that the cycle time of any user can be divided into two parts. Let us consider the example in Fig. 2, where user A's cycle time is divided into Part 1 and Part 2. Specifically,

1) Part 1 consists of all transmissions of user B and the first successful transmission of user A. We denote by n_B the number of successful transmissions of user B in Part 1.
2) Part 2 consists of all remaining successful transmissions of user A, and we denote its number by n'_A. Note that n'_A can be 0, in which case the duration of Part 2 is 0.

In Fig. 2, the duration of Part 1 and Part 2 is t_3-t_1 and t_5-t_3, respectively.

In CSMA/CA, users have the same distributions of the number of inter-transmissions, which can be denoted by a common random variable N_I. We define $P_{N_I,k}\triangleq\Pr(N_I=k)$. To ease exposition, we introduce the following notations:

- We use ℓ to represent the duration of a packet or a process, e.g., ℓ_{difs}, ℓ_{pkt}, ℓ_{ack}, ℓ_{rts}, ℓ_{cts}. We further define $\ell_{\text{tran}}\triangleq\ell_{\text{pkt}}+\ell_{\text{ack}}$, $\ell_{\text{rcts}}\triangleq\ell_{\text{rts}}+\ell_{\text{cts}}$, and $\ell_{\text{nav}}\triangleq\ell_{\text{tran}}+\ell_{\text{rcts}}-T_{\text{slot}}$, where T_{slot} is the slot time of CSMA/CA.
- For each user, the minimum contention window size is CW_{min}. After the i-th collision, the contention window size $\text{CW}_i=\min\{2^i\cdot\text{CW}_{\text{min}},\text{CW}_{\text{max}}\}$, where $\text{CW}_{\text{max}}=2^\beta\cdot\text{CW}_{\text{min}}$. The random backoff counter λ_i is uniformly chosen from $[1,\text{CW}_i]$, i.e., $\lambda_i\sim U(1,\text{CW}_i)$.

Lemma 5. *The duration of Part 1 and Part 2 is*

$$T_{part1} = n_B(\ell_{difs} + \ell_{nav}) + \ell_{tran} + \sum_{i=0}^{\rho_1}(\ell_{difs} + \ell_{rcts} + \lambda_i), \quad (11)$$

$$T_{part2} = \sum_{j=0}^{n'_A}\left[\ell_{tran} + \sum_{i=0}^{\rho_2^{(j)}}\left(\ell_{difs} + \ell_{rcts} + \lambda_i^{(j)}\right)\right], \quad (12)$$

where ρ_1 is the number of collisions in Part 1, $\rho_2^{(j)}$ is the number of collisions that occurred in Part 2 for the j-th transmission of user A, and $\lambda_i^{(j)}$ is the random backoff counter after the i-th collision for the j-th transmission of A. We emphasize that $\rho_2^{(j)}$ are independent and identically distributed (i.i.d.) random variables, $\forall j$.

Proof. The proof is very much involved, please see our technical report [21] for more details. ■

Theorem 6. *Consider a two-user multiple-access network. The CCT of CSMA/CA in the RTS/CTS mode is given by*

$$\Psi_{CSMA/CA} = \frac{1}{1-P_{N_I,0}}\left[\ell_{difs} + \ell_{nav} + \ell_{tran} + \frac{\ell_{difs}+\ell_{rcts}}{1-p_c} + \sum_{i=0}^{\beta-1} p_c^i \frac{1+2^i CW_{min}}{2} + \frac{p_c^\beta}{1-p_c}\frac{1+2^\beta CW_{min}}{2}\right], \quad (13)$$

where p_c is the collision probability of CSMA/CA that satisfies the following recursive equation with a unique solution:

$$p_c = \frac{2(1-2p_c)}{(1-2p_c)(CW_{min}+3) + p_c CW_{min}[1-(2p_c)^\beta]}. \quad (14)$$

Proof. See our technical report [21]. ■

The CCT of CSMA/CA in the basic model can be derived in a similar way, giving

$$\Psi_{\text{CSMA/CA-Basic}} = \frac{1}{1-P_{N_I,0}}\left[\ell_{\text{difs}} + \ell_{\text{tran}} - T_{\text{slot}} + \frac{\ell_{\text{difs}}+\ell_{\text{tran}}}{1-p_c} + \sum_{i=0}^{\beta-1} p_c^i \frac{1+2^i \text{CW}_{\text{min}}}{2} + \frac{p_c^\beta}{1-p_c}\frac{1+2^\beta \text{CW}_{\text{min}}}{2}\right]. \quad (15)$$

B. CCT-optimal CSMA/CA

Assuming $\text{CW}_{\text{max}} = \text{CW}_{\text{min}}$, our focus is on identifying the optimal CW value to optimize the CCT of CSMA/CA.

From (13), we have

$$\Psi_{\text{CSMA/CA}} = \frac{1}{1-P_{N_I,0}}\left[\frac{2(\ell_{\text{difs}}+\ell_{\text{rcts}})}{\text{CW}_{\text{min}}+1} + \frac{\text{CW}_{\text{min}}+1}{2} + C\right],$$

where $C \triangleq 2\ell_{\text{difs}} + \ell_{\text{nav}} + \ell_{\text{rcts}} + \ell_{\text{tran}} + 1$. It is easy to find that the minimum $\Psi^*_{\text{CSMA/CA}}$ is obtained when $\text{CW}_{\text{min}} = 2\sqrt{\ell_{\text{difs}} + \ell_{\text{rcts}}} - 1$.

On the other hand, [23] shows that, when $\text{CW}_{\text{max}} = \text{CW}_{\text{min}}$, the maximum throughput of CSMA/CA in a two-user multiple access network can be obtained when $\text{CW}_{\text{min}} = 2\sqrt{\ell_{\text{difs}} + \ell_{\text{tran}}} - 1$. Therefore, we have the same result as that in slotted Aloha: the parameters that achieve the minimum channel cycle time also give us the maximum throughput.

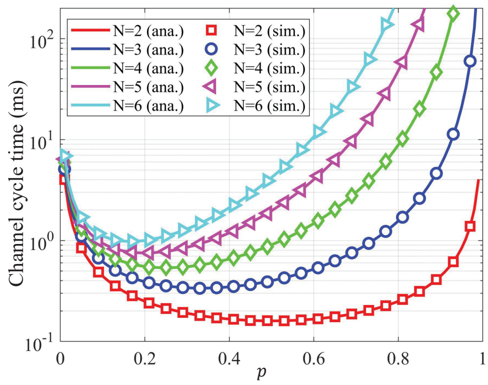

Fig. 3: CCT of a homogeneous network with slotted Aloha.

V. Simulation Results

This section presents analytical and simulation results to evaluate the CCT of slotted Aloha and CSMA/CA.

We first evaluate the CCT of slotted Aloha. To validate our derivations, we compare the analytical and simulation results of CCT versus the user transmission probability p under various user numbers N in Fig. 3 where we set packet duration to $T_{\text{slot}} = 20\mu$s. As can be seen, the simulation results are aligned with our analytical results very well for all cases.

For any user number N, CCT decreases first and then increases, as we increase p from 0 to 1. This observation matches our intuition: when p is small, there are not many packets in the channel, thus the packets can be transmitted more frequently as p increases, leading to a smaller CCT. On the other hand, as p becomes larger and larger, the collision probability increases, resulting in a waste of channel resources and an increase in the CCT. If CCT is used as the design principle to optimize slotted Aloha, the user transmission probability should be set to $p^* = 1/N$, which corroborates our analytical results in Section III.

Next, we compare the CCT of CSMA/CA and slotted Aloha, benchmarked against round-robin TDMA, which is known to be the short-term fairest. The results are presented in Fig. 4, where the duration of a slot, DIFS, ACK, RTS and CTS are set to $T_{\text{slot}} = 20\mu$s, $\ell_{\text{difs}} = 80\mu$s, $\ell_{\text{ack}} = 20\mu$s, $\ell_{\text{rts}} = 20\mu$s and $\ell_{\text{cts}} = 20\mu$s, respectively; the minimum and maximum CW sizes are $\text{CW}_{\text{min}} = 32T_{\text{slot}}$ and $\text{CW}_{\text{max}} = 1024T_{\text{slot}}$, respectively. As shown, CSMA/CA is a short-term fairer protocol than slotted Aloha. Moreover, the CCT of CSMA/CA exhibits a relatively fixed proportional gap with that of round-robin TDMA. The reason behind this is that, unlike slotted Aloha, the negotiation overhead of CSMA/CA is relatively constant to ℓ_{pkt}, which is also revealed by (13). Therefore, the slopes of the CCT of CSMA/CA and round-robin TDMA w.r.t. ℓ_{pkt} are on an equal footing.

Finally, we evaluate the CCT of CSMA/CA with more than two users in the basic and RTS/CTS modes. As can be seen, in both the basic and RTS/CTS modes of CSMA/CA, CCT increases almost linearly in the packet length ℓ_{pkt}. The CCT of CSMA/CA in the RTS/CTS mode is smaller than that in the basic mode when ℓ_{pkt} is large; while they can be larger when ℓ_{pkt} and N are small. Therefore, for large ℓ_{pkt}, the RTS/CTS

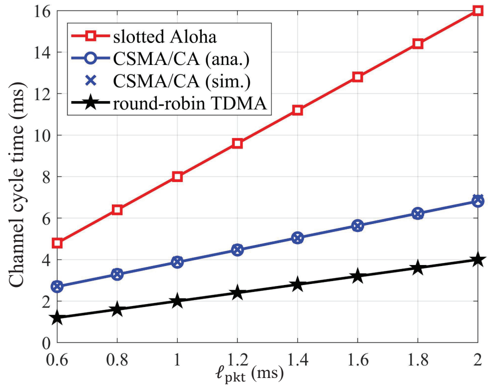

Fig. 4: Comparisons of channel cycle time versus ℓ_{pkt} for different MAC protocols.

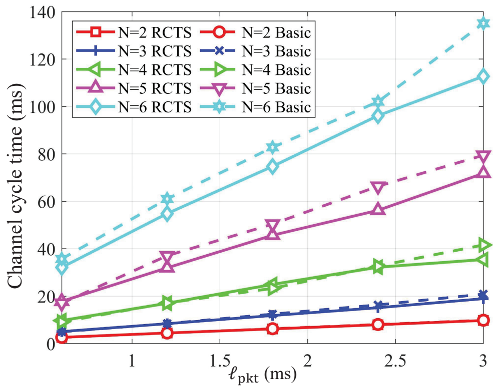

Fig. 5: CCT of CSMA/CA in the basic and RTS/CTS modes in a homogeneous network with more than two users.

mode is superior to the basic mode as far as the short-term fairness is concerned.

VI. Conclusion

Short-term fairness plays a crucial role in real-time applications. Conventional methods primarily focused on successful transmissions and employed a set of values or distributions to assess short-term fairness. In this paper, we introduced and thoroughly explored the concept of channel cycle time (CCT) as a metric for measuring short-term fairness in multiple-access networks. This metric, which characterizes the average duration between two successful transmissions of a user, during which all other users have successfully accessed the channel at least once, offers a fresh perspective on evaluating the transient behavior of MAC protocols. Moreover, CCT's emphasis on users' delay provides a more comprehensive view of short-term fairness, aligning with the evolving needs of modern networks.

The demonstrated effectiveness of CCT through the comparison of two classical MAC protocols, slotted Aloha and CSMA/CA, underscores its practical utility. The analytical derivation of closed-form CCT values reveals that CSMA/CA outperforms slotted Aloha in terms of short-term fairness, validating the metric's discriminatory power. Beyond its role as a metric, CCT can be used as a guiding principle in MAC protocol design. By strategically optimizing CCT during the development process, we devised MAC protocols for a two-user heterogeneous network that excel in short-term fairness.

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