Decentralized high-resolution direction finding in partly calibrated arrays

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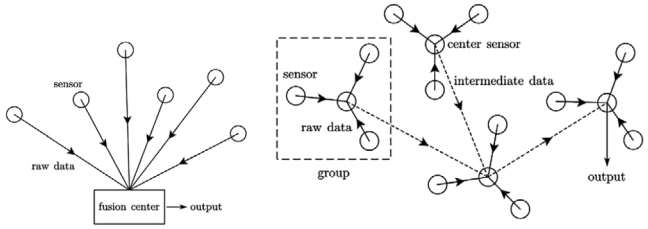
The problem of high-resolution direction finding is considered using a distributed array network with array position errors, called partly calibrated array. These errors are common in practice, and significantly affect high angular resolution. Existing calibration methods rely on centralized data transmission, which requires all the measurements to be transmitted to a computation center. This centralized mode leads to communication bottlenecks and system inflexibility. Here, a decentralized mode is designed, where each sensor only communicates locally with their neighboring sensors. Based on this mode, a corresponding direction-finding approach is proposed using sequential methods that process data one by one, which corresponds to the decentralized signal processing sensor by sensor. Simulations verify that the proposed method achieves decentralization with minor cost in estimation accuracy.

Introduction: We consider using distributed arrays to achieve high-resolution direction finding [1]. The key point of realizing high angular resolution lies in the coherent processing of the subarrays. However, in practice, there are inevitable position errors between the subarrays due to, for example, the motion of the subarray platform. These errors destroy the coherence between the subarrays, and challenge high angular resolution. Distributed arrays with position errors are referred to as partly calibrated array, which have been extensively studied.

To eliminate the effect of array position errors, various methods have been proposed, such as the subspace method RARE [2] and sparse recovery COBRAS [3]. However, these techniques rely on multiple snapshots and are not suitable for the scenarios with high real-time requirements and less data transmission. Hence, we proposed a direction finding method with only a single snapshot in [4]. This method uses blind source separation (BSS) to characterize the signal structure of partly calibrated arrays, and achieves high angular resolution, inversely proportional to the whole array aperture.

However, these approaches above are all in the centralized mode, that is, they require all the subarrays to transmit their measurements to a fusion center, where joint signal processing is implemented for direction finding. This centralized mode has the following drawbacks: (1) The fusion center is usually far from the sensor network, hence there are communication bottlenecks due to transmit power limitation, path loss and interference [5]. (2) The centralized system is not flexible and robust. If the fusion center breaks down, the whole system is affected. To this end, the idea of decentralized processing is proposed. In the decentralized mode, each sensor only communicates locally with their neighboring sensors to exchange the raw measurements or intermediate computation results, and signal processing is implemented iteratively in the network. In this way, we can solve the communication bottlenecks and improve the system's flexibility and robustness. For partly calibrated arrays, some decentralized direction finding methods have been proposed [5], but they are based on subspace methods and require many snapshots. In this letter, we focus on decentralized direction finding using partly calibrated arrays with only a single snapshot.

Our main contributions lie in two aspects: (1) We design a decentralized mode of data transmission. (2) We propose a corresponding method based on this mode. In the decentralized mode, we divide the sensor network into several small centralized groups, and transmit data within and between the groups. Particularly, within the groups, the raw measurements are transmitted to a sensor, called the center sensor. Then, through local signal processing, the center sensor outputs some intermediate results. These results are transmitted and computed between the center sensors of these groups until the last group, where the direction finding results are outputted. The centralized and decentralized modes are shown in Figure 1. The corresponding method is based on our previous centralized work [4]. Here, we design a decentralized setting of [4] to avoid communication bottlenecks and improve the system flexibility. The main idea of decentralization is to replace BSS with sequential BSS [6], since sequential ideas are to process data one by one, which matches with the decentralized signal processing sensor by sensor in partly calibrated arrays. The 'sensor by sensor' mode helps us avoid relying on a computation center for data fusion.



(a) Centralized mode. (b) Decentralized mode.

Fig. 1 *The centralized and decentralized modes*

The rest of the paper is organized as follows: We first introduce the signal model of partly calibrated arrays. Then, we review our previous centralized method, followed by proposing its decentralized implementation. Finally, numerical simulations verify the feasibility of our decentralized mode.

Signal model: Consider K linear antenna subarrays lying on a straight line. Each is composed of M elements and has the same manifold. Denote by $\eta_m \in \mathbb{R}$ the intra-subarray interval between the m-th element and the 1-st one, $m = 1, \ldots, M$. Denote by $\xi_k \in \mathbb{R}$ the inter-subarray displacement of the k-th subarray w.r.t. the 1-st subarray, $k = 1, \ldots, K$. The inter-subarray and intra-subarray displacement vectors are defined as $\boldsymbol{\xi} = [\xi_1, \ldots, \xi_K]^T$ and $\boldsymbol{\eta} = [\eta_1, \ldots, \eta_M]^T$, respectively. For partly calibrated arrays, $\boldsymbol{\eta}$ is perfectly known, but $\boldsymbol{\xi}$ is unknown. Assume that all the elements share a common sampling clock, and clocks of subarrays are synchronized.

Consider L far-field, narrow-band emitters impinging their signals onto the whole array from directions $\theta_1, \ldots, \theta_L \in (-\pi/2, \pi/2)$, $L \geqslant 1$. We define $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_L]^T \in \mathbb{R}^{L\times 1}$. We consider a single snapshot of the received signals, which are expressed as

$$\boldsymbol{y}_k = \sum_{l=1}^{L} x_l \boldsymbol{c}_k(\theta_l) + \boldsymbol{n}_k \in \mathbb{C}^{M\times 1}, \tag{1}$$

where $x_l \in \mathbb{C}$ is the complex amplitude of the l-th source, and $\boldsymbol{n}_k$ are the noises. We set $\boldsymbol{x} = [x_1, \ldots, x_L]^T \in \mathbb{C}^{L\times 1}$. The steering vector $\boldsymbol{c}_k(\theta)$ is expressed as

$$\boldsymbol{c}_k(\theta) = \boldsymbol{a}(\theta)\phi_k(\theta, \xi) \in \mathbb{C}^{M\times 1}, \tag{2}$$

where $\phi_k(\theta, \xi) = \exp(j\frac{2\pi}{\lambda}\xi_k \sin\theta)$ is the phase offset between the k-th and the 1-st subarrays, $\boldsymbol{a}(\theta)$ with $[\boldsymbol{a}(\theta)]_m = \exp(j\frac{2\pi}{\lambda}\eta_m \sin\theta)$ denotes the intra-subarray steer vector, and λ is the wavelength.

Stacking the measurement, steering and noise vectors above into matrices yield $\boldsymbol{Y} = [\boldsymbol{y}_1, \ldots, \boldsymbol{y}_K]$, $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_L)]$ and $\boldsymbol{N} = [\boldsymbol{n}_1, \ldots, \boldsymbol{n}_K]$, respectively. Adding the source amplitudes and the phase offsets into $\boldsymbol{X}$ with the (l, k)-th element given by $[\boldsymbol{X}]_{l,k} = x_l\phi_k(\theta_l, \xi)$, we rewrite (1) as

$$\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{N}, \tag{3}$$

where $\boldsymbol{A}$ is related to $\{\boldsymbol{\theta}, \boldsymbol{\eta}\}$, and $\boldsymbol{X}$ is related to $\{\boldsymbol{\theta}, \boldsymbol{\xi}, \boldsymbol{x}\}$. In (3), $\{\boldsymbol{Y}, \boldsymbol{\eta}, L\}$ is assumed to be known, and $\{\boldsymbol{\theta}, \boldsymbol{\xi}, \boldsymbol{x}, \boldsymbol{N}\}$ is unknown. Direction finding

using partly calibrated arrays is to recover $\boldsymbol{\theta}$ from $\boldsymbol{Y}$ in (3), which is a non-trivial problem since both $\boldsymbol{\theta}$ and $\boldsymbol{\xi}$ are unknown.

Review of the centralized method: We briefly review our method proposed previously in [4], called direction finding in partly calibrated arrays using BSS and matched filtering (MF), abbreviated as BSS-MF. It works in a centralized mode, and we will propose a sequential implementation in the next section. This method has two main steps: First estimate the phase offsets $\phi_k(\theta, \xi)$ between the subarrays, and then estimate the directions based on the phase offset estimates.

Phase offset estimation: The phase offsets are recovered by inputting the received signals $\boldsymbol{Y}$ in (3) to a BSS method named Joint Approximate Diagonalization of Eigen-matrices (JADE) [7], which then outputs the estimated $\boldsymbol{X}$ as $\hat{\boldsymbol{X}}$. The phase offsets $\phi_k(\theta_l, \xi)$ are recovered as $\hat{\phi}_{l,k}$, $\hat{\phi}_{l,k} = [\hat{\boldsymbol{X}}]_{l,k}/|[\hat{\boldsymbol{X}}]_{l,k}|$.

Direction finding: Based on $\hat{\phi}_{l,k}$, directions are estimated by MF as

$$\hat{\theta}_l = \underset{\theta\in(-\frac{\pi}{2},\frac{\pi}{2})}{\operatorname{argmax}} \left| \sum_{k=1}^{K} \boldsymbol{y}_k^H \left(\boldsymbol{a}(\theta)\hat{\phi}_{l,k} \right) \right|. \tag{4}$$

In [4], we show that BSS-MF achieves high angular resolution, inversely proportional to the whole array aperture.

The key idea of BSS-MF is the application of JADE, which aims to recover $\boldsymbol{x}(t) \in \mathbb{C}^{L\times 1}$ from $\boldsymbol{y}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{n}(t)$ with unknown $\boldsymbol{A} \in \mathbb{C}^{N\times L}$ and noises $\boldsymbol{n}(t)$. When the components of $\boldsymbol{x}(t)$ are independent ($\mathbb{E}[\boldsymbol{x}(t)\boldsymbol{x}^H(t)] = \boldsymbol{I}$), $\boldsymbol{x}(t)$ is recoverable by solving an optimization problem with the following steps:

1) Whiten: $\boldsymbol{z}(t) \equiv \boldsymbol{W}\boldsymbol{y}(t)$, where the whitening matrix $\boldsymbol{W} \in \mathbb{C}^{L\times N}$ is given by $\boldsymbol{W} \equiv [\boldsymbol{\Sigma}_y^T - \hat{\sigma}\boldsymbol{I}]_{1:L}^{-1/2}\boldsymbol{U}_y^H$, such that $\mathbb{E}[\boldsymbol{z}(t)\boldsymbol{z}^H(t)] = \boldsymbol{I}$. Here, $\boldsymbol{R}_y \equiv \mathbb{E}[\boldsymbol{y}(t)\boldsymbol{y}^H(t)] = \boldsymbol{U}_y\boldsymbol{\Sigma}_y\boldsymbol{U}_y^H$ is the singular value decomposition (SVD) of $\boldsymbol{R}_y$, $[\boldsymbol{\Sigma}_y^T]_{1:L} \in \mathbb{C}^{L\times N}$ denotes the first L nonzero rows of $\boldsymbol{\Sigma}_y$, $\cdot^{-1/2}$ denotes element-wise operation over the nonzero entries, and $\hat{\sigma} = \frac{1}{N-L}\sum_{l=L+1}^{N}[\boldsymbol{\Sigma}_y]_{l,l}$ is the noise power, estimated as the average of the $N-L$ smallest eigenvalues of $\boldsymbol{R}_y$.
2) Find a unitary matrix $\boldsymbol{V} \in \mathbb{C}^{L\times L}$ maximizing the independence level between components in $\boldsymbol{V}^H\boldsymbol{z}(t)$; see [7] for details.
3) Output: $\hat{\boldsymbol{x}}(t) = \boldsymbol{V}^H\boldsymbol{W}\boldsymbol{y}(t)$.

The proposed method: Here, we propose a decentralized implementation of the previous BSS-MF method. The main idea is:

I) Realize the BSS with a sequential mode [6].
II) Realize MF with local data within a group of subarrays.

Particularly, we divide the subarrays into N groups, and denote the sets of subarray indices by $\mathcal{K}_1, \ldots, \mathcal{K}_N$. We have $\cup_n\mathcal{K}_n = \{1, \ldots, K\}$. Without loss of generalization, we assume that the information is conveyed from the 1st group to the 2nd and finally to the N-th group. We first perform the sequential BSS as follows:

I.1) Locally whiten: Calculate the whitening matrix $\boldsymbol{W}_n \in \mathbb{C}^{L\times M}$, and the data $\boldsymbol{z}_k = \boldsymbol{W}_n\boldsymbol{y}_k$ individually for each group. This step requires the measurements $\boldsymbol{y}_k$ collected in each group, $k \in \mathcal{K}_n$. Then, the SVD of the covariance matrix of $[\ldots, \boldsymbol{y}_k, \ldots]$, $k \in \mathcal{K}_n$ is carried out, where each measurement corresponds to a subarray in the n-th group.
I.2) Find the unitary matrix $\boldsymbol{V}$ sequentially. The n-th group calculates the update matrix

$$\boldsymbol{H}_n \equiv \boldsymbol{H}_{n-1} \cdot \Pi_{k\in\mathcal{K}_n}\boldsymbol{G}_k \in \mathbb{C}^{L\times L}, n = 1, \ldots, N, \tag{5}$$

and transmits it to next group, where $\boldsymbol{H}_0 \equiv \boldsymbol{I}$ and $\boldsymbol{G}_k \equiv \boldsymbol{I} + \mu(\boldsymbol{I} - \boldsymbol{Q}_k/\|\boldsymbol{Q}_k\|) \in \mathbb{C}^{L\times L}$ with μ being the step size, $\boldsymbol{Q}_k \equiv \boldsymbol{f}(\boldsymbol{z}_k)\boldsymbol{z}_k^H$ and $\boldsymbol{f}(\cdot)$ being an element-wise function with specific statistical significance [6], usually selected empirically, $k \in \mathcal{K}_n$.
In the final group, the unitary matrix is given by $\boldsymbol{V}^N = \boldsymbol{H}_N\boldsymbol{V}^0$, where $\boldsymbol{V}^0 \equiv \boldsymbol{I}$. This strategy is obtained from the sequential BSS method [6], which calculates the unitary matrix $\boldsymbol{V} \in \mathbb{C}^{L\times L}$ with the measurement from each subarray one by one, given by

$$\boldsymbol{V}^{(k+1)} = \boldsymbol{V}^{(k)} + \mu(\boldsymbol{I} - \boldsymbol{Q}_k/\|\boldsymbol{Q}_k\|)\boldsymbol{V}^{(k)}, \tag{6}$$

where k denotes the index of the subarray.
I.3) Output: $\hat{\boldsymbol{x}}_k = \boldsymbol{V}^N\boldsymbol{W}_N\boldsymbol{y}_k$ for $k \in \mathcal{K}_N$.
II) Local MF: Finally, estimate the local phase offsets by

$$\hat{\phi}_{l,k} = \frac{[\hat{\boldsymbol{x}}_k]_l}{\|[\hat{\boldsymbol{x}}_k]_l\|}, l = 1, \ldots, L, k \in \mathcal{K}_N, \tag{7}$$

and then the directions using local MF, that is,(4) with $k \in \mathcal{K}_N$ instead of $k \in \{1, \ldots, K\}$.

Algorithm 1 Decentralized BSS-MF

Input: The received signals $\boldsymbol{Y}$, the intra-subarray displacement $\boldsymbol{\eta}$, the sensor index sets $\mathcal{K}_1, \ldots, \mathcal{K}_N$, and the emitter number L.
1) Local whitening as $\boldsymbol{z}_k = \boldsymbol{W}_n\boldsymbol{y}_k$, $k \in \mathcal{K}_n$.
2) Implement the iteration (6) as Figure 1b.
3) Local phase offset estimation as (7).
4) Estimate $\boldsymbol{\theta}$ by local MF as (4) with $k \in \mathcal{K}_N$.

Output: Direction estimation $\hat{\theta}_l$ for $l = 1, \ldots, L$.

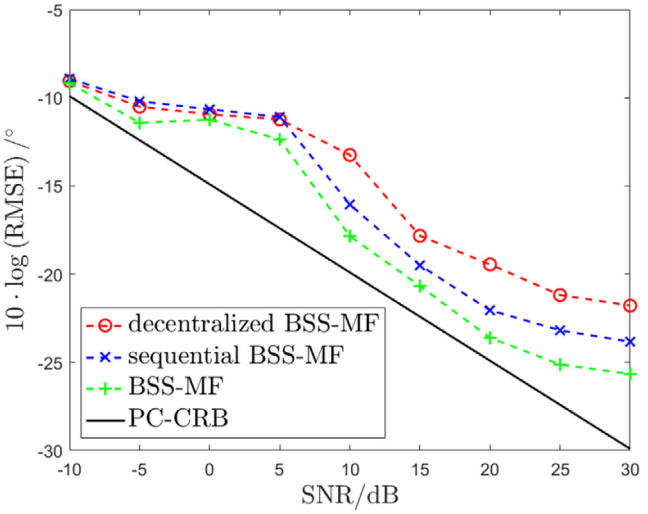


Fig. 2 *RMSE of methods w.r.t. SNR*

This method, denoted by decentralized BSS-MF, is summarized in Algorithm 1. BSS-MF is viewed as decentralized BSS-MF with $N = 1$. The complexity of decentralized BSS-MF is divided into sequential BSS and MF realization. The former is mostly composed of SVD, which complexity is $O(M^2 \times K)$, and the later is $O(M \times |\mathcal{K}_N| \times N_s)$, where $|\cdot|$ denotes the cardinality and N_s is the number of search times. In summary, the complexity is about $O(M^2 \times K + M \times |\mathcal{K}_N| \times N_s)$.

Simulation: Here we present simulation results. Consider $L = 2$ emitters in the directions $\boldsymbol{\theta} = [1.2, 1.6]^\circ$. There are $K = 20$ half-wavelength uniform linear subarrays with $M = 20$ elements. The inter-subarray displacement is set as $\xi_k = \frac{(k-1)D}{K-1}$, where $D = 450\lambda$ is the whole array aperture corresponding to the angular resolution $\lambda/D \approx 0.13^\circ$. Noises $\boldsymbol{N}$ are i.i.d. white Gaussian with mean $\boldsymbol{0}$ and variance $\sigma^2\boldsymbol{I}$. Here signal-to-noise ratio (SNR) is defined as $1/\sigma^2$. In the decentralized mode, we set $N = 3$, $|\mathcal{K}_1| = |\mathcal{K}_2| = 6$, $|\mathcal{K}_3| = 8$ and $\mu = 0.25$. The element-wise function $\boldsymbol{f}(\cdot)$ in (6) is empirically selected as $f(x) = x^3$.

Denote by sequential BSS-MF the centralized BSS-MF that only replaces JADE with sequential BSS [6]. We compare the root mean square error (RMSE) of methods with Cramér-Rao lower bound (CRB), denoted PC-CRB. The comparison results are shown in Figure 2. Note that the RMSEs of sequential BSS-MF are slightly larger than BSS-MF, yielding that JADE has better performance in phase offset estimation. The performance of decentralized BSS-MF is slightly worse than sequential BSS-MF, because decentralized BSS-MF implements local

whitening and MF within the group, but sequential BSS-MF uses the whole data. The small RMSE difference implies that we can achieve a more flexible decentralized system at a minor cost of estimation accuracy.

Conclusion: Our main contribution is to propose a decentralized mode of direction finding in partly calibrated arrays. We believe that a wide application of sequential techniques on decentralization is a potential direction. Lower communication and complexity are also required.

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